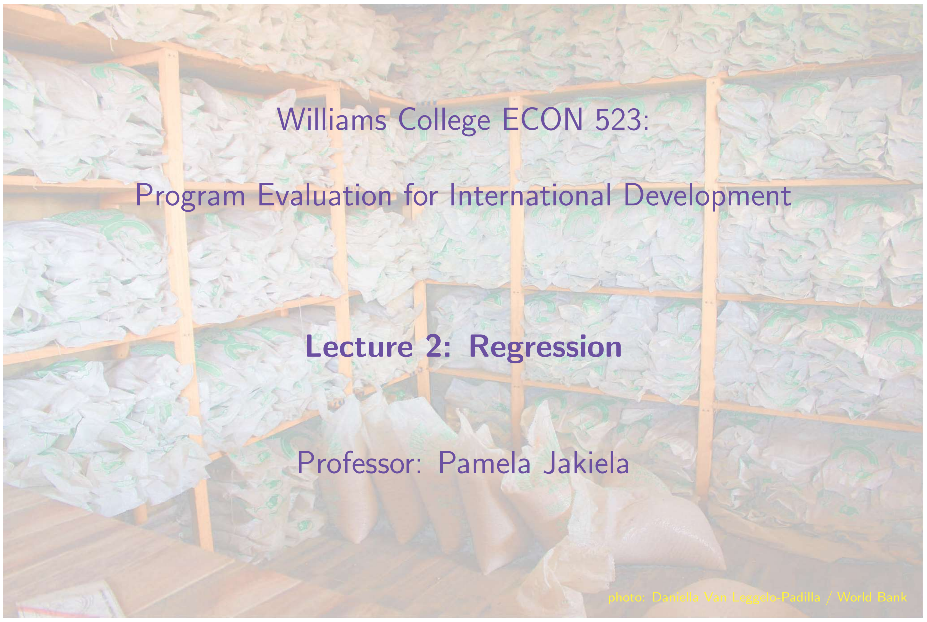


Williams College ECON 523:

Program Evaluation for International Development

Lecture 2: Regression

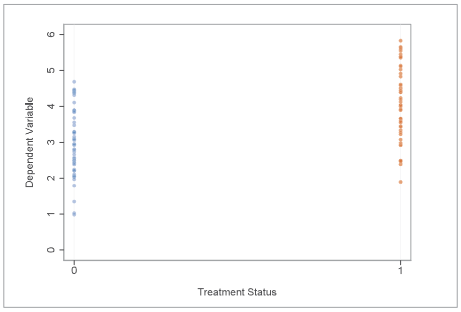
Professor: Pamela Jakiela

photo: Daniella Van Leggelo-Padilla / World Bank

Dummy Variables

OLS Regression on a Binary Independent Variable

$$Y = \alpha + \beta D$$



control	treatment
$D = 0$	$D = 1$
$\hat{\alpha}$	$\hat{\alpha} + \hat{\beta}$

$\hat{\alpha} = \bar{Y}_C$ (control group mean)

$\hat{\beta} = \bar{Y}_T - \bar{Y}_C$ (difference in means)

OLS Regression on a Binary Independent Variable

You may or may not remember that in a bivariate regression:

$$\hat{\beta}_{OLS} = \frac{COV(X,Y)}{VAR(X)} = \frac{\sum_i (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_i (X_i - \bar{X})^2}$$

Notice that the numerator can be re-organized:

$$\sum_i (X_i - \bar{X})(Y_i - \bar{Y}) = \sum_i X_i Y_i - \sum_i \bar{X} Y_i$$

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$$\sum_i (X_i - \bar{X})(Y_i - \bar{Y}) = \sum_i X_i Y_i - \sum_i \bar{X} Y_i$$
$$= \sum_i [Y_i (X_i - \bar{X})]$$

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When independent variable is binary:

$\bar{X} = \frac{n_T}{N}$ (n_T is # of treated observations)

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When independent variable is binary:

$\bar{X} = \frac{n_T}{N}$ (n_T is # of treated observations)

Assume observations are ordered:

$$\{\underbrace{Y_1, Y_2, \ldots, Y_{n_{T-1}}, Y_{n_T}}_{\text{treatment group}}, \underbrace{Y_{n_{T+1}}, Y_{n_{T+2}}, \ldots, Y_N}_{\text{control group}}\}$$

treatment group → $X_i = 1$ $\Rightarrow X_i - \bar{X} = 1 - \bar{X}$

control group → $X_i = 0$ $\Rightarrow X_i - \bar{X} = -\bar{X}$

OLS Regression on a Binary Independent Variable

You may or may not remember that in a bivariate regression:

$$\hat{\beta}_{OLS} = \frac{COV(X,Y)}{VAR(X)} = \frac{\sum_i [Y_i(X_i - \bar{X})]}{\sum_i (X_i - \bar{X})^2} = \frac{\sum_i [Y_i(X_i - \bar{X})]}{N\bar{X}(1 - \bar{X})}$$

Re-write denominator:

$$\sum_i (X_i - \bar{X})^2 = \sum_{i=1}^{n_T} (1 - \bar{X})^2 + \sum_{i=n_{T+1}}^{N} (-\bar{X})^2$$
$$= n_T (1 - \bar{X})^2 + (N - n_T)(-\bar{X})^2$$
$$= \ldots = n_T - n_T\bar{X} = N\bar{X}(1 - \bar{X})$$

OLS Regression on a Binary Independent Variable

You may or may not remember that in a bivariate regression:

$$\hat{\beta}_{OLS} = \frac{COV(X,Y)}{VAR(X)} \qquad \sum_i \left[Y_i \left(X_i - \bar{X}\right)\right]$$

$$= \frac{\sum_i \left[Y_i \left(X_i - \bar{X}\right)\right]}{\sum_i \left(X_i - \bar{X}\right)^2}$$

$$= \frac{\sum_i \left[Y_i \left(X_i - \bar{X}\right)\right]}{N\bar{X}\left(1 - \bar{X}\right)}$$

"linear combination of Ys"

OLS Regression on a Binary Independent Variable

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$$\hat{\beta}_{OLS} = \frac{COV(X,Y)}{VAR(X)} \qquad \sum_i \left[Y_i \left(X_i - \bar{X}\right)\right]$$

$$= \frac{\sum_i \left[Y_i \left(X_i - \bar{X}\right)\right]}{\sum_i \left(X_i - \bar{X}\right)^2} \quad = \sum_{i=1}^{n_T} \left[Y_i \left(1 - \bar{X}\right)\right] + \sum_{i=n_{T+1}}^{N} \left[Y_i \left(-\bar{X}\right)\right]$$

$$= \frac{\sum_i \left[Y_i \left(X_i - \bar{X}\right)\right]}{N\bar{X}\left(1 - \bar{X}\right)} \quad = \sum_{i=1}^{n_T} Y_i - \sum_{i=1}^{N} \left[Y_i \left(\bar{X}\right)\right]$$

$$= n_T \bar{Y}_T - \bar{X}\left(N\bar{Y}\right)$$

$$= N\bar{X}\bar{Y}_T - N\bar{X}\left[\bar{X}\bar{Y}_T + \left(1 - \bar{X}\right)\bar{Y}_C\right]$$

$$= N\bar{X}\left(1 - \bar{X}\right)\left(\bar{Y}_T - \bar{Y}_C\right)$$

OLS Regression on a Binary Independent Variable

You may or may not remember that in a bivariate regression:

$$
\begin{aligned}
\hat{\beta}_{OLS} &= \frac{COV(X,Y)}{VAR(X)} \\
&= \frac{\sum_i \left[Y_i \left(X_i - \bar{X}\right)\right]}{\sum_i \left(X_i - \bar{X}\right)^2} \\
&= \frac{\sum_i \left[Y_i \left(X_i - \bar{X}\right)\right]}{N\bar{X}\left(1 - \bar{X}\right)} \\
&= \frac{N\bar{X}\left(1-\bar{X}\right)\left(\bar{Y}_T - \bar{Y}_C\right)}{N\bar{X}\left(1-\bar{X}\right)}
\end{aligned}
$$

$$
\begin{aligned}
& \textstyle\sum_i \left[Y_i \left(X_i - \bar{X}\right)\right] \\
&= \textstyle\sum_{i=1}^{n_T} \left[Y_i \left(1 - \bar{X}\right)\right] + \sum_{i=n_{T+1}}^{N} \left[Y_i \left(-\bar{X}\right)\right] \\
&= \textstyle\sum_{i=1}^{n_T} Y_i - \sum_{i=1}^{N} \left[Y_i \left(\bar{X}\right)\right] \\
&= n_T \bar{Y}_T - \bar{X}\left(N\bar{Y}\right) \\
&= N\bar{X}\bar{Y}_T - N\bar{X}\left[\bar{X}\bar{Y}_T + \left(1 - \bar{X}\right)\bar{Y}_C\right] \\
&= N\bar{X}\left(1 - \bar{X}\right)\left(\bar{Y}_T - \bar{Y}_C\right)
\end{aligned}
$$

OLS Regression on a Binary Independent Variable

$$
\begin{aligned}
\hat{\beta}_{OLS} &= \frac{COV(X,Y)}{VAR(X)} \\
&= \frac{\sum_i \left[Y_i \left(X_i - \bar{X}\right)\right]}{\sum_i \left(X_i - \bar{X}\right)^2} \\
&= \frac{\sum_i \left[Y_i \left(X_i - \bar{X}\right)\right]}{N\bar{X}\left(1 - \bar{X}\right)} \\
&= \frac{N\bar{X}\left(1-\bar{X}\right)\left(\bar{Y}_T - \bar{Y}_C\right)}{N\bar{X}\left(1-\bar{X}\right)} \\
&= \bar{Y}_T - \bar{Y}_C
\end{aligned}
$$

OLS Regression on a Binary Independent Variable

When we regress Y_i on (only) a dummy variable:

$$\hat{\beta}_{OLS} = \bar{Y}_T - \bar{Y}_C$$

- Estimated constant $\hat{\alpha}_{OLS}$ is control group mean, also $\hat{Y}_i$
- Predicted $\hat{Y}_i$ for treated individuals/units is $\hat{\alpha}_{OLS} + \hat{\beta}_{OLS}$

OLS Regression on Mutually Exclusive Dummy Variables

$$Y = \alpha + \beta_1 T_1 + \beta_2 T_2 + \beta_3 T_3$$

control	treatment 1	treatment 2	treatment 3
$T_1 = T_2 = T_3 = 0$			

OLS Regression on Mutually Exclusive Dummy Variables

$$Y = \alpha + \beta_1 T_1 + \beta_2 T_2 + \beta_3 T_3$$

control	treatment 1	treatment 2	treatment 3
$T_1 = T_2 = T_3 = 0$	$T_1 = 1$	$T_2 = 1$	$T_3 = 1$
$\hat{\alpha}$	$\hat{\alpha} + \hat{\beta}_1$	$\hat{\alpha} + \hat{\beta}_2$	$\hat{\alpha} + \hat{\beta}_3$

$\hat{\alpha} = \bar{Y}_C$ (control group mean)

$\hat{\beta}_i = \bar{Y}_{T_i} - \bar{Y}_C$ (difference in means between treatment i and control)

Pooling Treatments

- If we pool treatments to estimate an average effect across treatment arms:
  - Estimated coefficient (and treatment effect) β_{pooled} is average of impacts across treatments
  - Average depends on n_{T_i} values: share of treated observations in each treatment arm
  - Also depends on treatment effect of each arm (pooling arms with no impact will matter)
- Estimates of pooled effect more precise because sample size is larger
  - When is pooled effect policy relevant?

Cross-Cutting Designs

- We often want to estimate the impact of treatments that may work best together
  - Access to credit and vocational training for unemployed youth
  - Nutrition supplements and parenting education for at-risk babies and children
  - Increased enforcement and information campaigns or behavioral nudges for tax compliance
  - Teacher training and additional materials for under-performing schools
  - Management consulting and subsidies for exporting firms
- Cross-cutting designs allow us to estimate impacts of each treatment in isolation as well as the pooled impact, to see whether any program effects are additively separable

Cross-Cutting Designs

	control	**treatment 2**
control	C	T2
treatment 1	T1	T1×T2

Cross-Cutting Designs and Interaction Terms

$$Y = \alpha + \beta_1 T_1 + \beta_2 T_2 + \beta_3 (T_1 \times T_2)$$

C	T2
T1	T1×T2

Challenge Problem: Triple Interactions

$$Y = \alpha + \beta_1 T_1 + \beta_2 T_2 + \beta_3 T_3 + \gamma_1 T_1 \times T_2 + \gamma_2 T_2 \times T_3 + \gamma_3 T_1 \times T_3 + \theta T_1 \times T_2 \times T_3$$

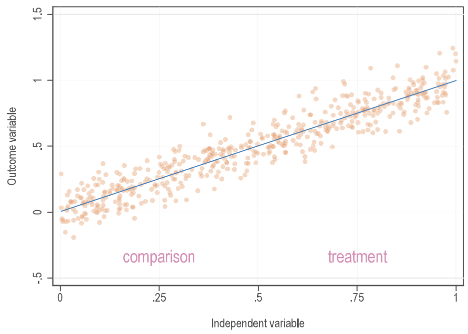
Continuous Variables

When Is Treatment a Continuous Variable?

- Sometimes we vary treatment intensity across treatment arms
  - Subsidies for malaria treatment (Cohen, Dupas, and Schaner 2015)
  - Varying the size of grants to entrepreneurs/firms, schools, etc.
  - Proportion treated within clusters (CCTs, job training, etc.)
- Binary treatments might also impact units differently, based on pre-existing conditions
  - Law banning traditional birth attendants in Malawi (Godlonton and Okeke 2016)
  - Impact of eliminating primary school fees on completion (cf. Lucas and Mbiti 2012)
- Should we dichotomize treatment variable or exploit continuous variation in intensity?

Continuous Variation in Treatment

$Y_i = X_i + \varepsilon$

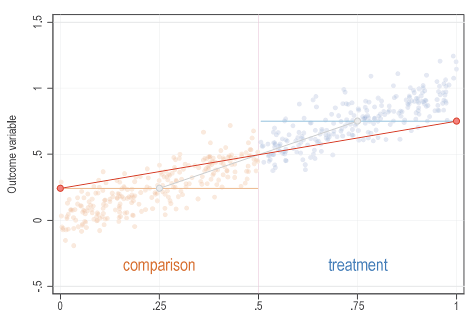


	(1)
	OLS
Treatment Intensity	0.994***
	(0.016)
Constant	0.005
	(0.009)

Standard errors in parentheses.

Continuous Variation in Treatment

$Y_i = X_i + \varepsilon$



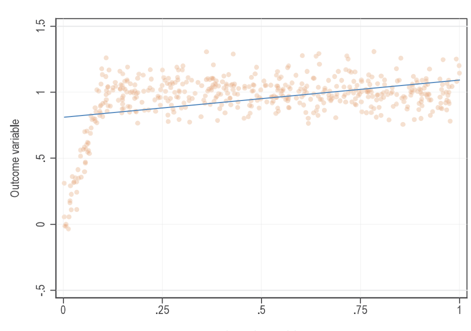
	(1)
	OLS
Treatment Dummy	0.509***
	(0.016)
Constant	0.242***
	(0.011)

Standard errors in parentheses.

Dichotomous vs. Continuous Treatment Variables

- When the true dose-response relationship is linear:
  - OLS w/ a continuous treatment variable is correctly specified
  - Uses observed variation increase statistical power (odds of finding an effect)
- The estimand is different when we dichotomize treatment
  - OLS coefficient captures impact of moving from average level of treatment intensity in the control group to average level of treatment intensity in the treatment group
    - Not the same as impact of moving from treatment intensity 0 to treatment intensity 1
- When true dose-response relationship is linear, OLS with continuous X is optimal

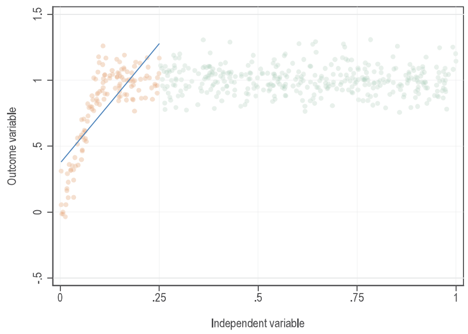
OLS when the Dose-Response Relationship Is Not Linear



	(1) OLS
Treatment Intensity	0.281***
	(0.029)
Constant	0.810***
	(0.017)

Standard errors in parentheses.

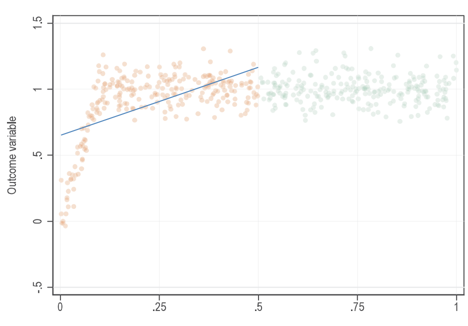
OLS when the Dose-Response Relationship Is Not Linear



	(1) OLS
Treatment Intensity	3.614***
	(0.257)
Constant	0.372***
	(0.034)

Standard errors in parentheses.

OLS when the Dose-Response Relationship Is Not Linear



	(1) OLS
Treatment Intensity	1.032***
	(0.092)
Constant	0.650***
	(0.026)

Standard errors in parentheses.

Continuous Variation in Treatment Intensity

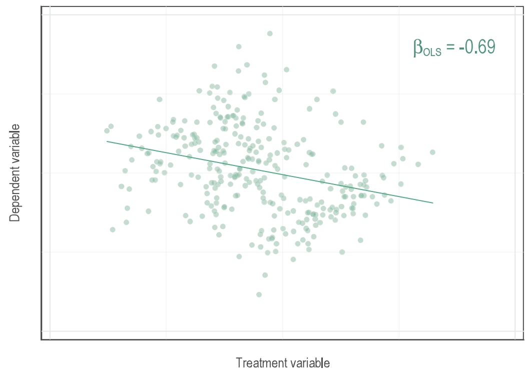
- When the true relationship (i.e. conditional expectation function) is **not** linear:
  - OLS w/ a continuous treatment variable is incorrectly specified (may or may not matter)
  - Estimated treatment effect (i.e. coefficient) depends on choice of sample (values of X)
- Graph your data (though often true relationship obscured by noise)
  - Choose to dichotomize (and where to dichotomize)
  - Vary your sample to assess the robustness of your estimates
- Be skeptical of results when treatment assignment process is unclear (observational data!) and you cannot observe the relevant empirical relationships in your data graphically

Fixed Effects

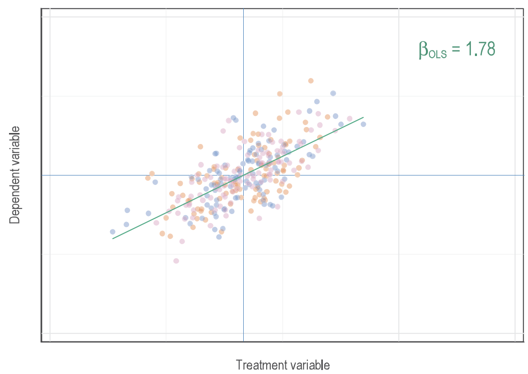
What Are Fixed Effects?

- Individual dummy variables for mutually exclusive groups in your data
  - Dummy for male or female
  - Age (or age group) fixed effets
  - Continent/country/state/district fixed effects
  - Year fixed effects
- Why use fixed effects?
  - Estimation using **within** rather than **between** variation
- We often use multiple sets of fixed effects in empirical work

Simpson's Paradox



What Do Fixed Effects Do?



What Do Fixed Effects Do?

- Fixed effects are equivalent to:
  - Transforming both independent and dependent variables by subtracting off the mean in each group and adding back the mean in the omitted category (the blue group in the figure)
    - Equivalently: subtract off the difference in means between group and omitted group
  - Run OLS in your transformed (i.e. re-centered) data
- Fixed effects mattered because treatment varied across groups
  - When treatment doesn't vary, FEs can improve precision but won't change slope estimate
  - When randomized/exogenous treatment probability/intensity varies across groups, you must include fixed effects or control directly for probability of treatment (the propensity score)
- If you regress X and Y on FEs, de-meaned variables are the residuals

The Frisch-Waugh-Lovell Theorem

$$Y = \alpha + \beta X + \gamma Z$$

is equivalent to

$$\tilde{Y} = \alpha + \beta \tilde{X}$$

where

$\tilde{Y}$ = residuals from regressing Y on Z

$\tilde{X}$ = residuals from regressing X on Z

Frisch-Waugh-Lovell in Practice

- These approaches generate identical estimates of β_{educ}, the estimated impact of education:
  1. Regress income on education, age dummies, and a dummy for being female
  2. Transform income and education by subtracting off age-specific means, then transform the transformed variables by subtracting off gender-specific means (of transformed variables), then regress transformed income on transformed education
  3. Generate residualized income and education by regressing those variables on the age and gender FEs, then regress residualized income on residualized education (without controls)
  4. Residualize income and education on age dummies first, then gender dummies, then regress

Frisch-Waugh-Lovell: Why It Matters

- When treatment is binary and plausibly exogenous, the difference in outcome means between the treatment and comparison groups provides an unbiased estimate of impact
  - All treated observations get equal positive weight, all untreated get equal negative weight
- With controls that are correlated with treatment, treatment is (in effect) no longer binary
  - Untreated observations with covariates that predict a high likelihood of treatment get very low negative weights in linear regression; while treated observations with covariates that predict a low likelihood treatment get very high positive weights in multivariate regression
  - Everything is still fine if treatment effects are homogeneous: treatment effect is the same for everyone, so it doesn't matter what weights we use to calculate average treatment effect
  - If effects vary with covariates that predict treatment, mis-specification problems arise
- What to do: show results with and without covariates, residualize and then plot your data

Empirical Exercise