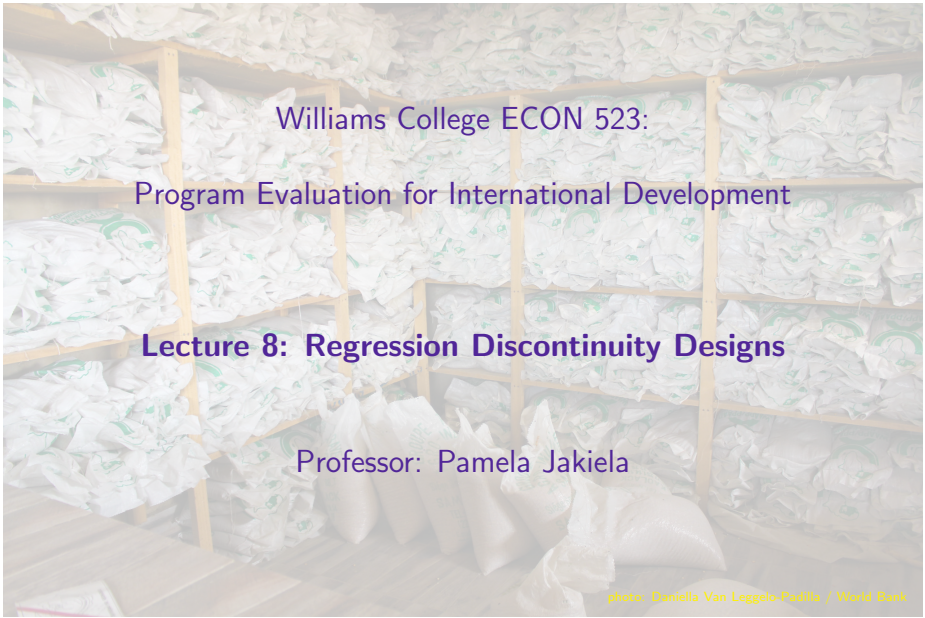


Williams College ECON 523:

Program Evaluation for International Development

Lecture 8: Regression Discontinuity Designs

Professor: Pamela Jakiela

photo: Daniella Van Leggelo-Padilla / World Bank

Regression Discontinuity: Intuition

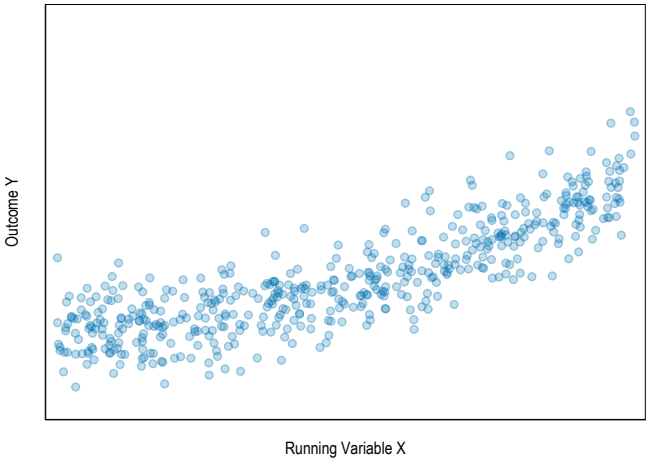
What Is a Regression Discontinuity?

In a **regression discontinuity** (RD) design: treatment status is determined by a precise rule based on a threshold value of a continuous characteristic (the running or forcing variable)

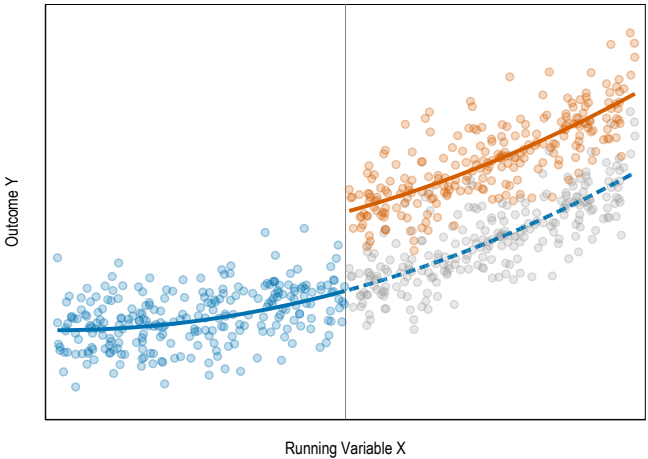
A few examples of potential RDs in development economics:

- **Academic test scores**: admission or scholarships awarded to those with a certain score (e.g. above the threshold for passing), or based on a precise percentile cutoff
- **Household income**: earned-income tax credit for those with incomes below cutoff
- **Poverty indices**: anti-poverty programs are often "proxy-means-tested"
- **Landholdings**: microfinance available to households with less than an acre of land
- **Date of birth**: eligibility for government programs such as preschool/kindergarten, medicare, vaccines, etc. often depends on whether you are above or below an age cutoff
- **Elections**: candidate who gets most (or more than fifty percent of the) votes wins

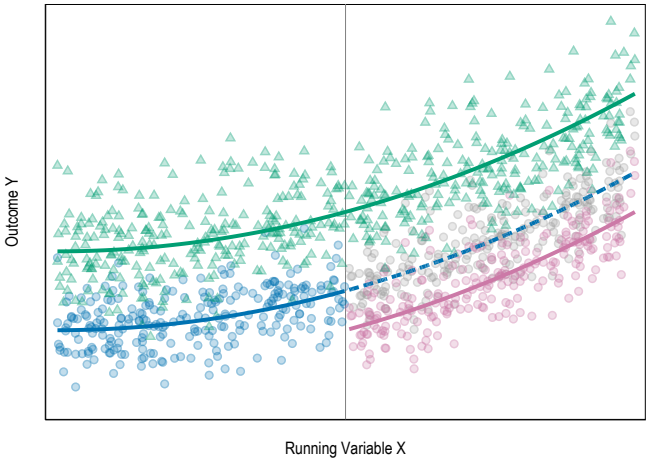
Y_0 Is a Continuous Function of Running Variable X



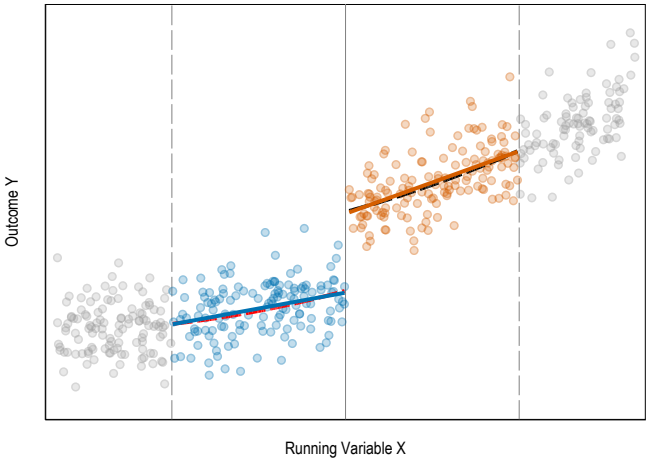
A Jump in $f(X)$ at the Discontinuity Indicates a Treatment Effect



Analogous to Diff-in-Diff, But Without a Comparison Group



With a Narrow Bandwidth, $f(X)$ Should Be Approximately Linear



When Can You Use RD?

RD designs are possible when eligibility is determined by some smooth continuous index, but participants don't have precise control over the index (so they can't select into treatment)

- Individuals with low vs. high values of the eligibility index may look very different, but differences should be quite small in the neighborhood right around the discontinuity
- Individuals might be able to influence the likelihood of being **near** the discontinuity, but those near the discontinuity can't make sure that they end up on one side vs. the other
  - Near discontinuity, treatment is "as good as random"

As in a randomized trial, you either have a valid discontinuity or you don't

When Can You Use RD?

For a valid RD, the following must be true:

1. Treatment status determined by a continuous variable (treated $\Leftrightarrow$ if above/below cutoff)
   - Eligibility index must be smooth, take many values around cutoff
   - Estimated treatment effect is limit as you approach cutoff
2. Individuals/units/etc. cannot **precisely** manipulate eligibility index
   - Precise manipulation $\Rightarrow$ selection bias
3. No other programs/treatments can use the same discontinuity

Implementing RD

RD Specification

RD setup:

- Let X_i denote the running variable
- Observations with X_i above cutoff C are treated, those below are not
- Define a re-centered running variable $Z_i = X_i - C$
  - Let T_i be an indicator for having $Z_i \geq 0$, and hence being treated

The most common empirical specification for a regression discontinuity design is:

$$Y_i = \alpha + \beta T_i + \theta_1 Z_i + \theta_2 (Z_i \times T_i) + \varepsilon_i$$

RD Specification

RD setup:

- Let X_i denote the running variable
- Observations with X_i above cutoff C are treated, those below are not
- Define a re-centered running variable $Z_i = X_i - C$
  - Let T_i be an indicator for having $Z_i \geq 0$, and hence being treated

The most common empirical specification for a regression discontinuity design is:

$$Y_i = \alpha + \beta T_i + \theta_1 Z_i + \theta_2 (Z_i \times T_i) + \varepsilon_i$$

Notice that we allow for separate slopes above and below the cutoff

Alternative RD Specifications

Quadratic RD specification includes linear, squared distance from the cutoff (above and below)

$$Y_i = \alpha + \beta T_i + \theta_1 Z_i + \theta_2 (Z_i \times T_i) + \lambda_1 Z_i^2 + \lambda_2 (Z_i \times T_i)^2 + \varepsilon_i$$

Adding additional polynomial terms allows for better fit of $Y = f(X)$

- Including higher order polynomial terms also leads to over-fitting, so we usually don't
- All continuous, differentiable functions are locally linear

Do make sure you allow $f(X)$ to differ above vs. below the discontinuity

Bandwidth Selection

Bandwidth selection implies a sample restriction

- Example: `reg y treatment z_below_cutoff z_above_cutoff if abs(z)` $\leq$ `bw, r`
- What is `bw`? How wide of a window around the cutoff is included in the analysis?
  - Bias: $f(X)$ is locally linear, but (probably) not linear; forcing linearity introduces bias
  - Variance: a narrow bandwidth means a small(er) sample, and hence a noisier estimate

Traditionally, most common approach to bandwidth selection is ad hoc: what looks good?

- Choose a range of data small enough such that relationship **looks** linear (in a graph)
- Compare results as you vary the bandwidth, from too small to too large

Bandwidth Selection: Examples

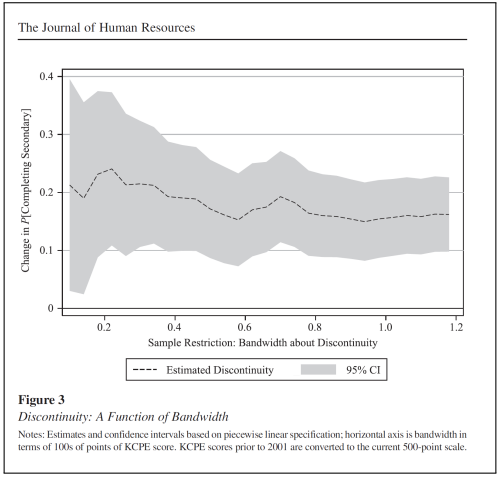


Figure 3

Discontinuity: A Function of Bandwidth

Notes: Estimates and confidence intervals based on piecewise linear specification; horizontal axis is bandwidth in terms of 100s of points of KCPE score. KCPE scores prior to 2001 are converted to the current 500-point scale.

Source: Ozier (2018)

Bandwidth Selection: Examples

TABLE III

ALTERNATIVE RD SPECIFICATIONS[a]

	Bandwidth				
	1	0.5	0.25	0.1	0.05
	(1)	(2)	(3)	(4)	(5)
Panel A: Women					
Polynomial order of control function					
None	0.012**	0.015**	0.018***	0.025***	0.018*
	(0.006)	(0.006)	(0.006)	(0.007)	(0.010)
Linear	0.014**	0.021***	0.025***	0.028**	0.039**
	(0.007)	(0.006)	(0.007)	(0.012)	(0.019)
Quadratic	0.027***	0.030***	0.033***	0.032*	0.051
	(0.007)	(0.007)	(0.010)	(0.018)	(0.032)
Cubic	0.031***	0.026***	0.036**	0.057**	0.054
	(0.007)	(0.010)	(0.015)	(0.028)	(0.042)
Quartic	0.030***	0.032**	0.044**	0.067**	0.028
	(0.009)	(0.012)	(0.017)	(0.033)	(0.056)
Observations	2628	2177	1049	489	257

Source: Myersson (2014)

Optimal Bandwidth Selection

More recent approaches aim to optimize bias-variance tradeoff

- Imbens and Kalyanaraman (2012): bandwidth used in Meyersson (2014)
- Calonico, Catteneo, and Titiunik (2014)
- Calonico, Catteneo, Farrell, and Titiunik (2017): `rdrobust` command in Stata/R

There's Always a Graph

Common elements of regression discontinuity graphs:

- The running variable is on the x-axis, and the cutoff is indicated
- Observations are binned and bin-level averages are shown in a scatter plot (**bin scatter**)
  - Bins either contain fixed N or fixed range of X values
- The continuous relationship between the running variable and the outcome is shown, always with separate graphs for above vs. below the cutoff for treatment eligibility
  - Sometimes a linear fit and sometimes a local-polynomial regression (`lpoly`)
  - Sometimes with confidence intervals, and sometimes without

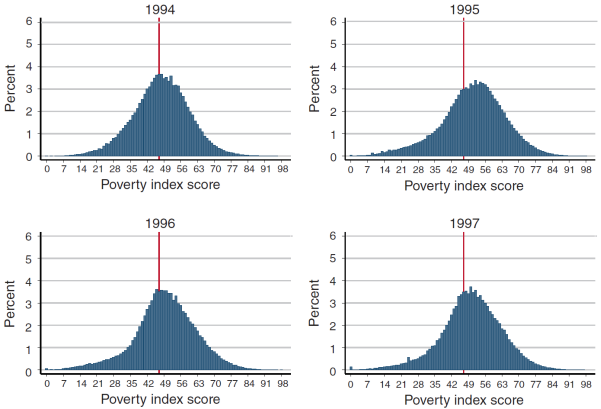
Fuzzy RD

In some cases, being above the discontinuity increases treatment, but not from 0 to 1

- An RD with imperfect compliance is a **fuzzy RD**
- Being above the cutoff is a valid instrument for treatment (controlling for X)
  - First stage: RD regression of treatment on cutoff, slopes above and below
  - Second stage: RD regression of outcome on predicted treatment, again with slope controls

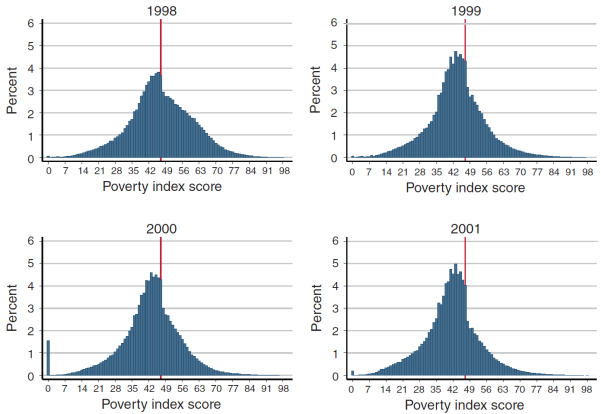
Assessing the Validity of an RD

Q: When Is an RD Not an RD?



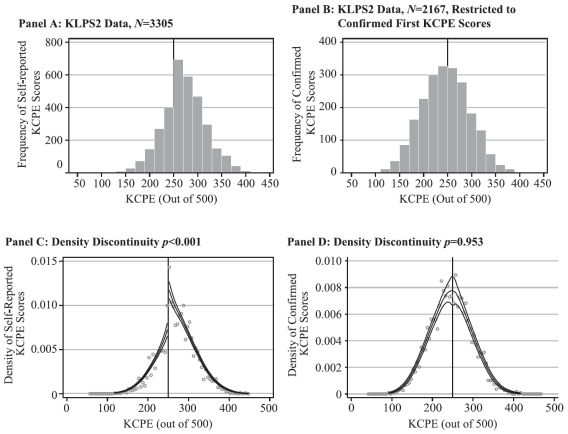
Source: Conover and Comacho (2011)

A: After Potential Participants Learn the Allocation Rule



Source: Conover and Comacho (2011)

A: Or, Potentially, When Scores Are Self-Reported



Source: Ozier (2018)

Defending the Validity of an RD Design

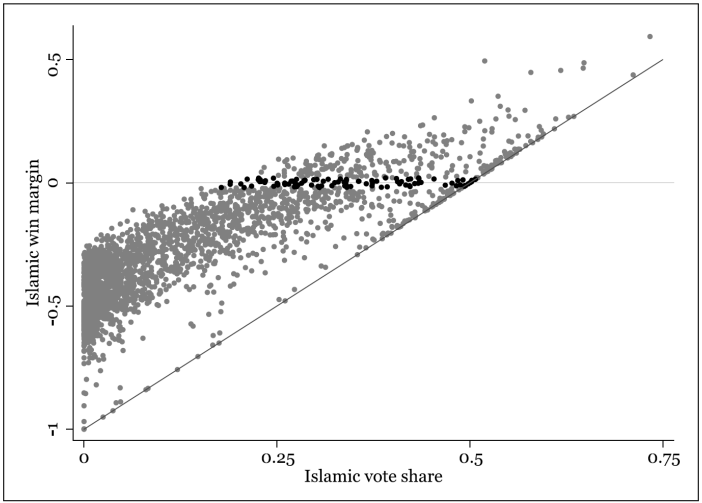
The assumption of no sorting/manipulation can't be tested directly

- Plot the distribution (McCrary (2007) also proposes a test)
- Plot the covariates: pre-treatment characteristics shouldn't jump with treatment

Two approaches empirical approaches:

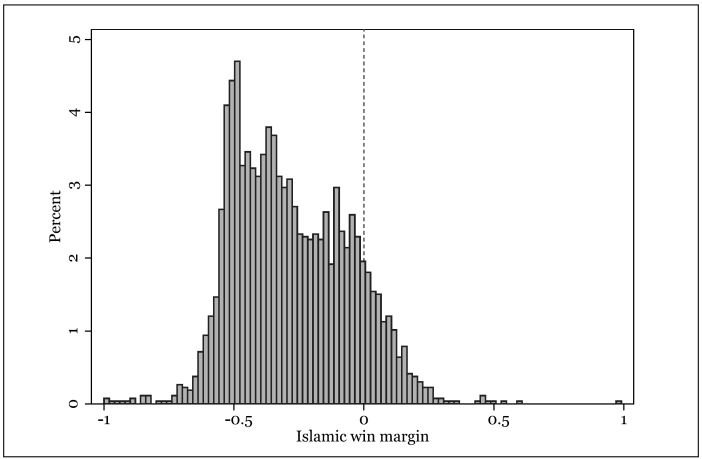
- Histogram (of either running variable or covariates)
- Local polynomial regression above vs. below (using `lpoly` command in Stata)

Checking for Manipulation



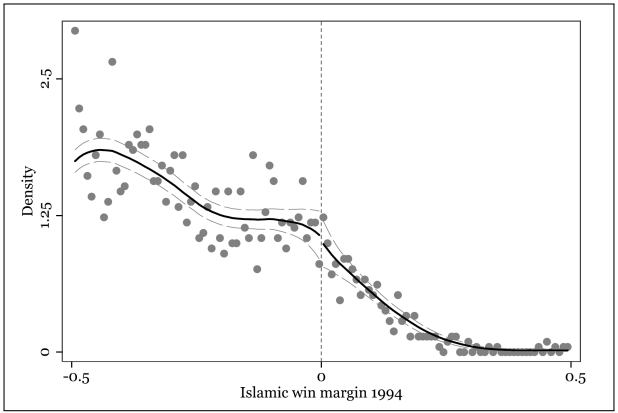
Source: Meyersson (2014)

Checking for Manipulation: Histogram



Source: Meyersson (2014)

Checking for Manipulation: Density Plot



Source: Meyersson (2014)

Checking Covariate Balance

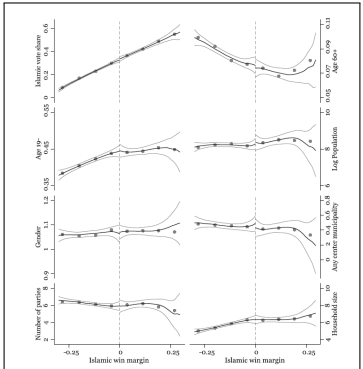


FIGURE 3.—Balanced covariate checks. The panels refer to the following covariates: Islamic vote share, population share above 60, population share below 19, log population, the gender ratio, a dummy for a center municipality, the number of vote-receiving parties, and household size. Each dot corresponds to the unconditional mean in bins of 8 percent by the Islamic win margin in mayoral elections 1994. The solid line represents the predicted values of a local linear smoother estimated using raw data on each side of the threshold at zero. Outer gray lines indicate 95 percent confidence intervals. A SUR test of the coefficients joint significance results in a p-value of 0.87.

Source: Meyersson (2014)

RD Checklist

1. Do you have an RD?
   - 1.1 Program eligibility determined by continuous running variable
   - 1.2 Many observations close to the discontinuity
   - 1.3 No manipulation
2. Bin scatter showing relationship between running variable and treatment probability
3. Show that running variable and covariate densities look smooth at cutoff
4. Choose your bandwidth and run your regressions
   - 4.1 Report 2SLS for fuzzy RD designs
   - 4.2 Show alternative bandwidths, higher-order polynomials as robustness checks