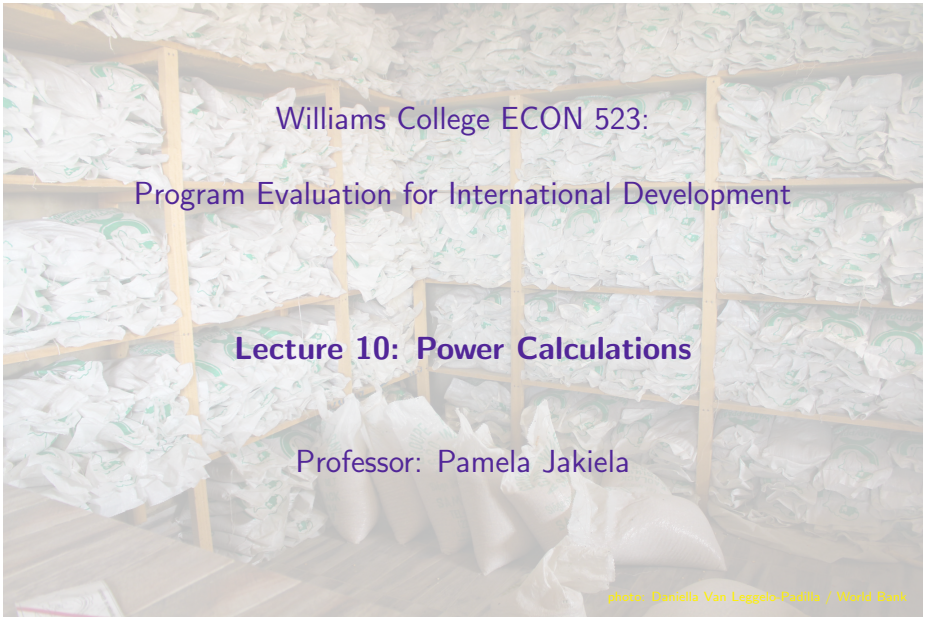


Williams College ECON 523:

Program Evaluation for International Development

Lecture 10: Power Calculations

Professor: Pamela Jakiela

photo: Daniella Van Leggelo-Padilla / World Bank

Hypothesis Testing

Every research design involves:

1. A null hypothesis and either one or several alternative hypotheses
2. A statistic related to our hypotheses that we can calculate from our data
3. A rule mapping values of our statistic into decisions about whether to reject the null

Type I and Type II Errors

Any statistic is a random variable:

- We don't know for sure that the statistic we calculate will be very close to the "true" value
- If we draw a random sample, the sample mean could be far from the population mean
- If we randomly assign treatment, most of the male/female/rural/urban/young/old/etc people could happen to end up in the treatment group rather than the control group

Our hypothesis testing procedure could lead to a mistake:

- We might see what looks like a large impact when there is no impact (Type I error)
- We might see what looks like no impact when there actually is an impact (Type II error)

Power Calculations and Sample Size

When we do **power calculations**, we are:

1. Choosing a maximum acceptable probability of Type I error, called our **test size**
   - We always choose $\alpha = 0.05$ (ever since Fisher)
2. Choosing a minimum acceptable probability of avoiding a Type II error, called our **power**
3. Figuring out how large a sample we need to achieve (1) and (2), which will depend on:
   - The properties of our outcome variable and chosen test statistic
   - Our beliefs about the likely impact of our policy intervention

Type I Errors and Test Size: Example

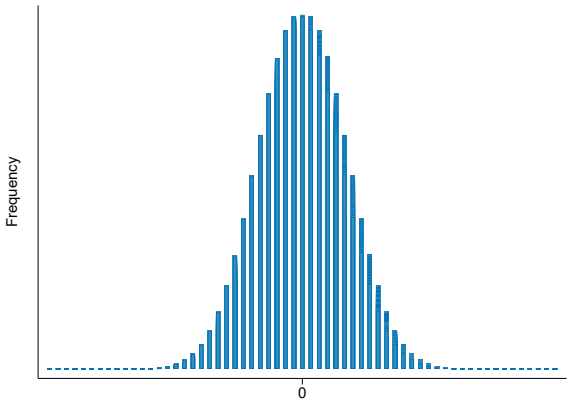
Randomly assign treatment in a sample of 1,000 people so that 500 are treatment, 500 control

- There are a lot of different possible random assignments
- Sometimes more tall people end up in treatment (or comparison) group

Test the hypothesis that average height is higher in treatment than in control

- Remember: we haven't implemented any type of treatment
- Average heights should be similar in both groups, on average
- Difference in means could still be large, depending on treatment assignment

Type I Errors and Test Size: Example

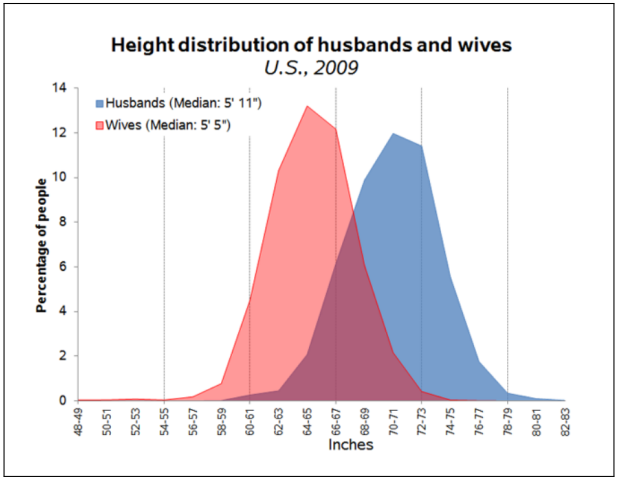


Test Size: The Probability of a Type I Error

The standard practice is to reject the null hypothesis when our test statistic translates into a p-value below 0.05, so statistics that "large" occur less than 5% of the time under the null

- Statistically significant differences occur by chance about 5% of the time
  - Why we stratify when randomly assigning treatment
  - Can lead to "publication bias" because findings get published (non-findings don't)
- Depends on sample size: larger sample $\Rightarrow$ smaller variance of chosen sample statistic
  - Test size is essentially fixed, but what $t = 1.96$ means in terms of outcome units depends on sample size, careful measurement of outcomes, ability to include meaningful control variables

Statistical Power



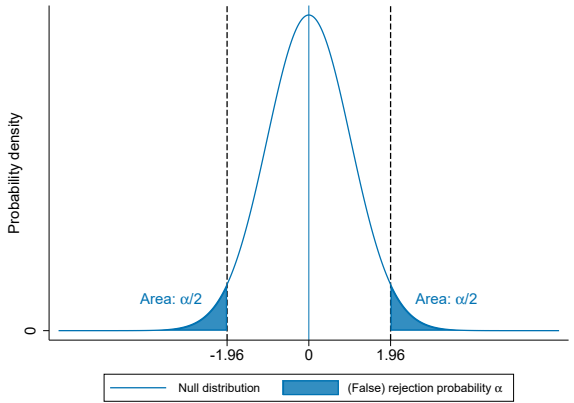
Source: Cohen (2013)

Statistical Power

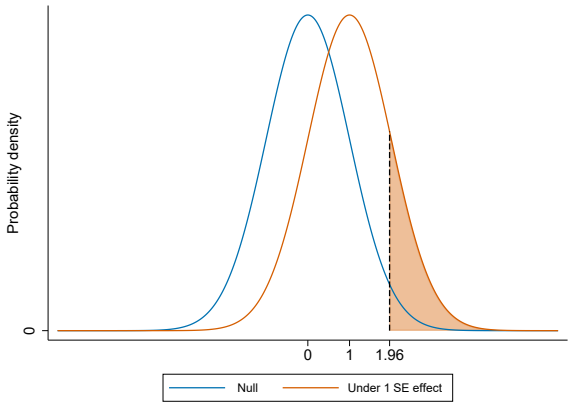
Wives (women) are shorter than husbands (men), on average, but distributions overlap

- If I measure the heights of N wives and N husbands, how likely is it that the difference in means will be large enough to reject the null hypothesis (given test size of $\alpha = 0.05$)?
- If I drew hundreds of different samples of N wives and N husbands, sometimes I'd see a large difference in average height, and sometimes I'd see a small difference in height
  - Estimated difference in height **when there is actually a difference** will also be normally distributed around the true (population) difference between male and female heights
  - If I knew the true difference in mean heights and its variance, I could tell you – for a fixed sample size – how often I'd expect to observe a statistically significant height difference
  - Depends on the variance of height in population, and on the sample size

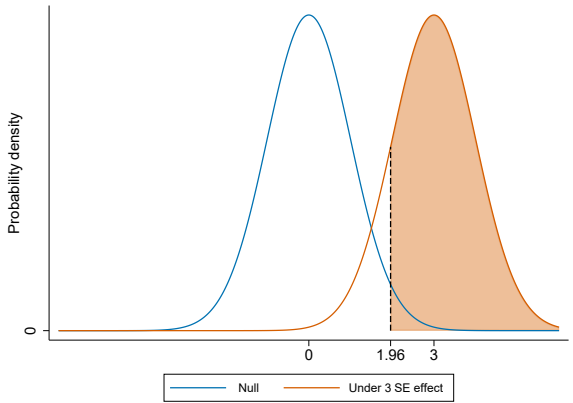
How Often Will We Reject the Null?



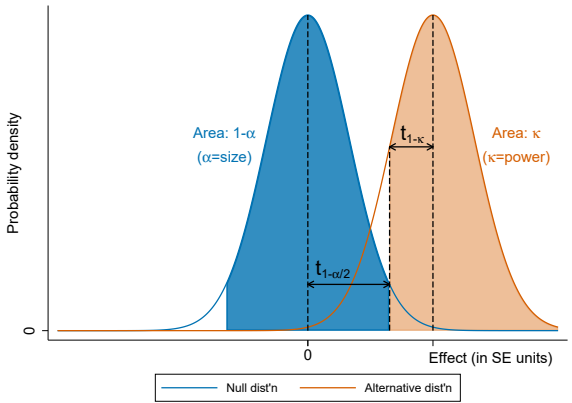
Power with a 1 SE Effect



Power with a 3 SE Effect



Statistical Power



The Minimum Detectable Effect

The **minimum detectable effect** (or MDE) is the smallest effect size that we can detect with power of 0.8 (i.e. the probability of a Type II error, failing to reject a false null, is 0.2)

$$\begin{aligned} \text{MDE} &= \left(t_{\alpha/2} + t_{1-\kappa}\right)\sqrt{\frac{1}{P(1-P)}}\sqrt{\frac{\sigma^2}{N}} \\ &\approx 2.8\sqrt{\frac{1}{P(1-P)}}\sqrt{\frac{\sigma^2}{N}} \end{aligned}$$

where:

- P is the proportion of the sample assigned to treatment
- N is the sample size
- σ^2 is the variance of the outcome

Implications of the Minimum Detectable Effect Formula

1. MDE is decreasing in N: a larger sample means great statistical power
2. MDE is maximized when $P = 1/2$ (though other factors such as costs may come into play)
3. MDE can be expressed in SD units, or converted into outcome variable units
   - Prior studies can suggest plausible effect sizes
   - Existing data sets can be used to calculate variance of some outcomes
   - For binary outcomes, variance only depends on mean and sample size

Practice Problems

1. You have a sample size of $N = 100$, but you can only offer the program to 20 people. Find the MDE in standard deviation units.
2. Half of your sample (of size N) is allocated to treatment and the other half to control. Find the sample size needed to have an MDE of 0.25 standard deviations.
3. You would like to know whether vocational training improves the profits of self-employed youth. Your main outcome of interest is income, which has a mean of 968 and a standard deviation of 11,842. If you assign half of your sample to treatment and half to control, how large of a sample would you need to have an MDE equivalent to 50% of average income?