# 11 Monopoly and Equilibrium

## 11.1 Aggregate Demand

- A simple model of demand when consumers are heterogeneous:
  - Population of $N$ consumer's indexed by $i$
  - Consumer $i$'s utility depends on two things:
    - The amount of money (dollars) that they hold, $d_i \in \boldsymbol{R}_+$
    - whether or not they own one unit of a(n indivisible) good, $x_i \in \{0, 1\}$
  - Consumer $i$'s valuation of the indivisible good $x$ is $v_i \in [0, v^{max}]$, and overall utility is given by:

$$u_i(d_i, x_i) = d_i + v_i x_i \tag{1}$$

    So, if $x_i = 0$, utility is simply the number of dollars that consumer $i$ has, $d_i$, but if $x_i = 1$, utility is the sum of dollars held and individual $i$'s valuation of the indivisible good.
  - We assume that $d_i$ is large enough that a consumer who does not have the good is able to purchase it. In other words, individuals are not "liquidity constrained."
  - Let $d_0$ denote initial wealth. Individual $i$ prefers to purchase the good at price $p > 0$ whenever:

$$d_0 - p + v_i \geq d_0 \Leftrightarrow v_i \geq p \tag{2}$$

    which does not depend on $d_0$ (so there is no need to worry about whether $d_0$ needs to be indexed by $i$).
  - Suppose individual valuations $v_i$ are uniformly distributed on the interval $[0, v^{max}]$. So $v_i$ can take on any value such that $0 \leq v_i \leq v_{max}$, and all values in that range are equally likely.
  - If this assumption is true, than for any possible price $p > 0$, the monopolist knows what fraction of the population will be willing to purchase the good, and

hence what total demand for the good will be as a function of price.

$$\begin{aligned}\Pr(\textsf{buy}) &= \Pr(v_i > p) \\ &= 1 - \Pr(v_i \leq p) \\ &= 1 - \frac{p}{v^{max}}\end{aligned} \tag{3}$$

– If there are $N$ people in the population, total demand for the indivisible good at price $p$ is

$$D(p) = N - \frac{Np}{v^{max}} \tag{4}$$

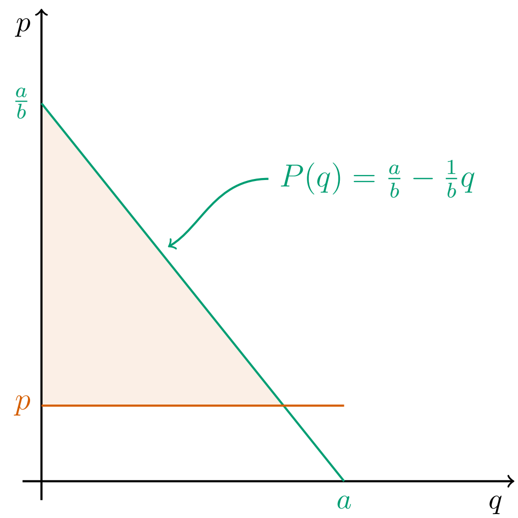
for $0 \leq p \leq v^{max}$; $D(p) = 0$ for $p > v^{max}$.

∘ Thus, we can think of the situation where demand is linear with $D(p) = a - bp$ as a way of representing an economy where each consumer decides whether to buy one unit of an indivisible good.

∘ When demand is linear, we can also invert the demand function to arrive at the **inverse demand function**:

$$P(q) = \frac{a}{b} - \left(\frac{1}{b}\right) q \tag{5}$$

where $P(q)$ indicates the market-clearing price if the producer wants to sell $q$ units of output.




- In this framework, individual utility is given by Equation 11, so $v_i - p$ is a measure of the welfare of any individual who purchases the good:

$$\begin{aligned} w_i &= u_i\,(\text{buy}) - u_i\,(\text{don't buy}) \\ &= d_0 - p + v_i - d_0 \\ &= v_i - p \end{aligned} \tag{6}$$

- The area between the demand curve and the price is thus a measure of overall (consumer) wellbeing which we term **consumer surplus**:

$$CS = \int_0^q [P(q) - p]\,dx \tag{7}$$

## 11.2 Monopoly

- A **monopolist** is the sole producer or provider of a good
- A **monopoly** is a marlet with only one producer or supplier
- In a monopoly, the producer decides how much to produce, but when she faces a downward-sloping demand curve, selling more means that she must charge a lower

price for each unit sold[1]

- A monopolist's revenues are:

$$R(q) = [P(q)]\, q \Rightarrow \frac{\partial R(q)}{\partial q} = P(q) + q \cdot \partial P(q) \partial q \tag{8}$$

- A monopolist differs from the price-taking producer we discussed in the previous unit: for a price-taker, marginal revenue is always equal to $p$
  - The second term in Equation 8 is absent because the price-taker does not need to lower the price of all inframarginal units in order to sell the one marginal unit

### 11.2.1 The Monopolist's Profit-Maximization Problem

- A profit-maximizing uniform-price monopolist solves

$$\max_{q \geq 0} [P(q)]\, q - c(q) \tag{9}$$

yielding the first-order condition:

$$P(q) + q \cdot \partial P(q) \partial q - \frac{\partial c(q)}{\partial q} = 0 \tag{10}$$

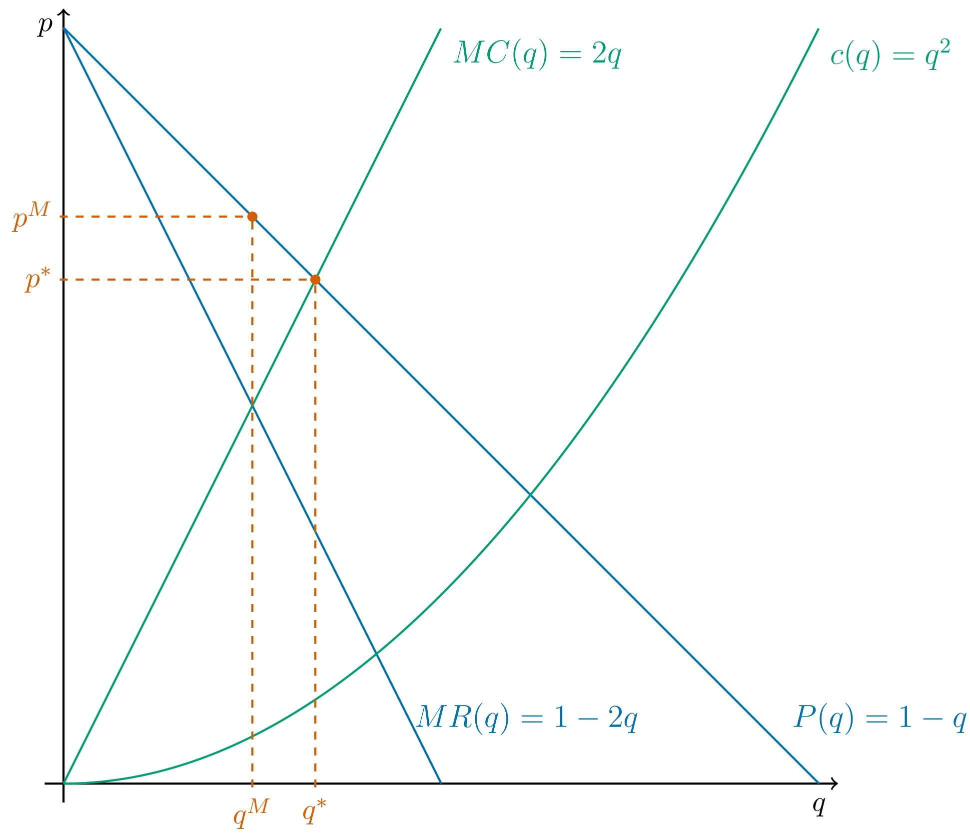
- A the profit-maximizing level of output, marginal revenue equals marginal cost
- **Example:** $P(q) = 1 - q$ and $c(q) = q^2$

$\Rightarrow$ $R(q) = q - q^2$

$\Rightarrow$ $\pi(q) = R(q) - c(q) = q - 2q^2$

$\Rightarrow$ $\frac{\partial \pi(q)}{\partial q} = 0 \Leftrightarrow 1 - 4q^* = 0 \Leftrightarrow q^* = \frac{1}{4}$

[1] Unless she can engage in some type of price discrimination, which we will discuss later.




- The profit-maximizing monopolist sets $q^M = 1/4$
- In contrast, a price-taking producer would set choose $q^*$ so that $p = MC(q^*)$, so the producer's marginal cost curve is her supply curve
- In this context, monopoly is **inefficient**: we can make some people better off without making anyone worse off (if we let the monopolist sell to the buyer with valuations between $p^M$ and $p^M$ without changing they price she charges the other buyers)
- This is what we refer to as the deadweight loss of monopoly (not just that the producer is getting more profits/surplus and the consumers are getting less)
- First-degree price discrimination
- Second and third degree price discrimination

## 11.3 Competitive Equilibrium

- Consider the indivisible good setup described before: consumer $i$'s valuation of an indivisible good $x$ is $v_i \in [0, v^{max}]$, and overall utility is given by:

$$u_i(d_i, x_i) = d_i + v_i x_i \tag{11}$$

  where $x_i$ is a dummy variable (either 0 or 1) indicating whether the consumer owns a single unit of the good

- There are 12 people in the economy: 6 potential sellers who own one unit of the good each, and 6 potential buyers who do not own a unit of the good

- The valuations of the potential sellers are:

| **ID** | $v_i$ |
|---|---|
| S1 | 1 |
| S2 | 1 |
| S3 | 2 |
| S4 | 3 |
| S5 | 5 |
| S6 | 8 |

- The valuations of the potential buyers are:

| **ID** | $v_i$ |
|---|---|
| B1 | 1 |
| B2 | 1 |
| B3 | 2 |
| B4 | 3 |
| B5 | 5 |
| B6 | 8 |

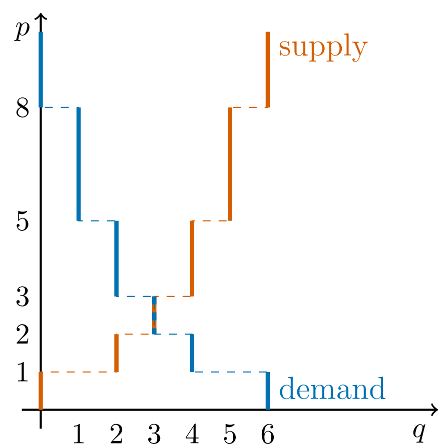
- In contrast to (most of) the models we've studied before, we now have a situation involving multiple people, and we'd like to make a prediction about what will happen when they interact

- A **competitive equilibrium** is a way that we can make a prediction about the result of an interaction between individuals or firms: we can predict that the market will be stable at a price, $p$ that "clears the market" such that demand equals supply

- Can we find an equilibrium price? Assume that if a buyer is indifferent, she buys, and if a seller is indifferent, she sells.

| **Price** | **Buyers** | **Sellers** |
|---|---|---|
| 1 | 6 | 2 |
| 2 | 4 | 3 |
| 2.5 | 3 | 3 |
| 3 | 3 | 4 |

- The price 2.50 clears the market, as would any price $2 < p < 3$




- The price that clears the market is where the supply curve intersects the demand curve

- In this example, there are equal numbers of potential buyers and potential sellers, and the distribution of valuations is the same in both groups

- This need not be the case: what price would clear the market if we added two additional potential buyers with valuations 13 and 21?

- In this setup, when there are $z$ units of the good available, the market clearing price must fall between valuations $v_z$ and $v_{z+1}$ if we ordered them

- The implication of this is that, in a competitive equilibrium, the individuals who most value the good will end up with it

- Is competitive equilibrium realistic?

- We haven't said anything about how these transactions take place: what if Buyer B6 walked into the mall and the first store he saw belonged to seller S1? What price might they agree on?

- One important requirement for competitive equilibrium is that no buyer or seller be large enough to influence the market price (unlike a monopolist)

- But we also need perfect information for all parties: there cannot be any uncertainty about the quality of the good, and buyers and sellers need to have a pretty good sense of the distribution of valuations unless there is an auctioneer or some other type of market maker

- Market experiments

## 11.4 Linear Supply and Linear Demand

- We've already derived conditions under which it is reasonable to represent market demand as $D(p) = a - bp$

- We can also represent aggregate supply as $S(p) = dp$

  - We can model heterogeneous agents, as in the case of demand, deciding whether or not to sell their single unit of an indivisible good

- Alternatively, if all producers are small and act as price-takers, we've seen that each producer's individual supply function is their marginal cost curve
- Aggregating multiple linear marginal cost functions (if the cost function itself is quadratic) will also result in a linear demand function that passes through the origin

◦ We can characterize the competitive equilibrium as the price that solves $D(p^*) = S(p^*)$, together with the resulting level of output, $q^*$