

ECON 251: Problem Set 9
Due Friday November 18 by 11:00 PM

Instructions. Each part of a problem is worth one point. Don't forget to answer the last two questions, as they are each worth one point. Unless otherwise stated, you can always assume that goods are continuously divisible: there is no need to consume an integer number of units of any good. Once you have finished, please submit your completed problem set on gradescope. To do this, you will either need to upload a pdf of your entire problem set or an image (for example, a picture that you take with your phone) of your work for each problem. If you upload a pdf, you will need to tag each problem on the appropriate page of the document. Please show your work and draw a box around your final answer. You are free to work together with your classmates, but the work that you upload must be your own.

1. Williamstown Widgets produces widgets that it sells for price $p = 100$. Williamstown Widgets' production function is

$$f(z) = 20 \ln(z + 1)$$

where $z \geq 0$ is the number of units of the input good used up in the production process. The price of unit of the input good is $w = 80$.

- (a) What is Williamstown Widgets' profit function?
 - (b) If Williamstown Widgets maximized profits, how many units of the input good will it choose to use?
2. Bill Bernoulli is the CEO and managing director of Bernoulli Brothers Mathematical Consulting. He employs his brothers, Ben and Bob Bernoulli, to prove mathematical theorems which Bill then sells for price $p = 24$. For each hour that one of his brothers works, Bill pays that brother wage $w = 8$ dollars, which is the hourly wage that either of the brothers would receive if he went to work outside the family firm. If Ben Bernoulli works $\ell_{ben} \geq 0$ hours proving theorems, he proves

$$f_{ben}(\ell_{ben}) = 3\sqrt{\ell_{ben}}$$

theorems. If Bob Bernoulli works $\ell_{bob} \geq 0$ hours proving theorems, he proves

$$f_{bob}(\ell_{bob}) = 4\sqrt{\ell_{bob}}$$

theorems. Bernoulli Brothers' revenues and profits depend on the total number of theorems proved by Ben and Bob, since all proved theorems are sold for the market price, $p = 24$

- (a) What is Bernoulli Brothers' profit function?
- (b) If Bernoulli Brothers' maximizes profits, how many hours do Ben and Bob work per week?
- (c) If Bernoulli Brothers' maximizes profits, what are total revenues?
- (d) If Bernoulli Brothers' maximizes profits, what are Bill Bernoulli's profits?
- (e) Bill Bernoulli only wants to operate Bernoulli brothers if total family income (i.e. Bernoulli Brothers' revenues) are at least as high as what Ben and Bob could make if they worked 36 hours per week outside the family firm, earning a wage of w dollars per hour (each). What is the maximum market wage at which Bill Bernoulli would be willing to operate the family firm?

3. Santa Claus produces toys using elf labor. He distributes his toys to children at no charge, but he receives a subsidy of $s > 0$ dollars per bag of toys from the Canadian government. He pays each elf a daily wage of $w > 0$. Santa's cost function is

$$c(q) = wq^2$$

indicating that it costs him wq^2 (Canadian) dollars to produce $q \geq 0$ bags of toys.

- (a) If Santa maximizes profits, how many bags of toys does he produce?
 - (b) If Santa maximizes **output** subject to the constraint that his profits cannot be negative, how many bags of toys does he produce?
 - (c) Because of new labor laws at the North Pole, Santa must hire elves on two-year contracts. This year, the subsidy is s per bag of toys and the wage rate is w per day of elf labor. Next year, Santa expects the subsidy to rise by 10 percent, but to keep up with increases in the cost of living, Santa will have to increase the elves' wage rate by 20 percent. Santa does not discount the future (being immortal and all). If Santa maximizes profits and he must use the same quantity of elf labor in each of the next two years, how many bags of toys does he choose to produce each year?
4. A profit-maximizing firm's cost function is $c(q) = a + bq^2$. It is paid p for every unit of output that it produces. What is the minimum price at which the firm will choose to produce a positive quantity of output?
5. Which of your classmates did you work with on this problem set?
6. Did you attend Jamie's TA office hours, or get help from her over email or outside of her regular office hours?