ECON 251: Problem Set 8
Due Friday November 11 by 11:00 PM

Instructions. Each part of a problem is worth one point. Don't forget to answer the last two questions, as they are each worth one point. Unless otherwise stated, you can always assume that goods are continuously divisible: there is no need to consume an integer number of units of any good. Once you have finished, please submit your completed problem set on gradescope. To do this, you will either need to upload a pdf of your entire problem set or an image (for example, a picture that you take with your phone) of your work for each problem. If you upload a pdf, you will need to tag each problem on the appropriate page of the document. Please show your work and draw a box around your final answer. You are free to work together with your classmates, but the work that you upload must be your own.

1. A discounted utility maximizer with the instantaneous utility function $u(x) = x$ and daily discount factor $\delta < 1$ is indifferent between receiving 512 dollars today and one thousand dollars in three days. What is her daily discount factor δ?

2. A discounted utility maximizer with the instantaneous utility function $u(x) = \sqrt{x}$ and daily discount factor $\delta = 0.98$ is indifferent between receiving ten thousand dollars today and x dollars per day every (from now until forever). What is x?

3. A discounted expected utility maximizer with the instantaneous utility function $u(x) = \ln x$ and daily discount factor $\delta < 1$ is indifferent between (a) receiving 1,024 dollars today and then one dollar every day forever and (b) playing the lottery $L = \left\{\frac{1}{3}, \frac{2}{3}; 8, 1\right\}$ every day forever. What is her daily discount factor δ?

4. A discounted utility maximizer with the instantaneous utility function $u(x) = \sqrt{x}$ and yearly discount factor $\delta < 1$ is indifferent between receiving one hundred dollars today and an annuity that pays her 100 dollars in one year and 400 dollars in two years. What is her yearly discount factor δ?

5. Studious Steve is working on an empirical research project that is due the day after tomorrow. He can work on it tonight and tomorrow night. He has to decide how many hours he will spend working on the project tonight, $h_0 \geq 0$, and how many hours he will spend working on it tomorrow night, $h_1 \geq 0$. If he works $H = h_0 + h_1$ hours in total, he will earn grade $100H$. (His professor says that while it is theoretically possible to earn a score above 100, for some reason it never happens.) Steve is a discounted utility maximizer with daily discount factor $\delta < 1$. His instantaneous utility function at time t is $G_t - h_t^2$ where G_t is a grade that he learns about on day t and h_t is hours of work on the project on day t.

   (a) Characterize the discounted utility function that Steve will seek to maximize at time $t = 0$ (which incorporates his disutility of effort and the utility he derives from the grade he receives).

   (b) What grade will he earn on the project (as a function of δ and t)?

6. Images Cinema is hosting a Wes Anderson Film Festival this week. They are showing Moonrise Kingdom on Tuesday, The Royal Tannenbaums on Wednesday, and Grand Budapest Hotel on Thursday. Everyone has the same view on the relative merits of these movies: seeing Moonrise Kingdom on day t yields instantaneous movie utility 3, seeing The Royal

Tannenbaums on day t yields instantaneous movie utility 5, and seeing Grand Budapest Hotel on day t yields instantaneous movie utility 8.

(a) Consistent Connie is deciding which of three movies she wants to see. She can only see one because she has homework that she needs to do two of the three nights. She is very patient, so her daily discount factor is $\delta = 1$. Which movie does she choose to see?

(b) Impatient Imogen is deciding which of three movies she wants to see. She can only see one because she has homework that she needs to do two of the three nights. She is very **im**patient, so her daily discount factor is $\delta = \frac{1}{2}$. Which movie does she choose to see?

(c) Sophisticated Sophie is deciding which of three movies she wants to see. She has daily discount factor $\delta = 1$, but she is present-biased: at any point in time, she discounts all future periods by the constant (not exponentiated) factor $\beta = \frac{1}{2}$. Sophisticated Sophie is aware of her present bias, so she knows her future self my not carry out the actions that her present self would prefer. If you asked Sohpie on Monday which one movie she would like to see, what would she say?

(d) If Sophisticated Sophie does not see Moonrise Kingdom on Tuesday, she will have to decide on Wednesday whether to see The Royal Tannenbaums on Wednesday or Grand Budapest Hotel on Thursday. Which movie will she choose to see?

(e) On Tuesday, Sophie is forward-looking enough to know the answer to (d): she knows which movie she will end up seeing if she does not go see Moonrise Kingdom on Tuesday. Given this knowledge, what does she decide to do on Tuesday? Which movie will she end up seeing?

(f) Like Sophie, Naive Niamh has daily discount factor $\delta = 1$, but she is present-biased: at any point in time, she discounts all future periods by the constant (not exponentiated) factor $\beta = \frac{1}{2}$. However, unlike Sophisticated Sophie, Naive Niamh is unaware of her present bias. She assumes that she will not discount the future in the future. If, on Monday or Tuesday, you asked her what she will do on Wednesday if she has not already gone to see a movie, what would she say?

(g) What will Naive Niamh decide to do on Tuesday? Which movie does she end up seeing?

7. Which of your classmates did you work with on this problem set?

8. Did you attend Jamie's TA office hours, or get help from her over email or outside of her regular office hours?