ECON 251: Problem Set 5

Due Friday October 14 by 11:00 PM

Instructions. Each part of a problem is worth one point. Don't forget to answer the last two questions, as they are each worth one point. Unless otherwise stated, you can always assume that goods are continuously divisible: there is no need to consume an integer number of units of any good. Once you have finished, please submit your completed problem set on gradescope. To do this, you will either need to upload a pdf of your entire problem set or an image (for example, a picture that you take with your phone) of your work for each problem. If you upload a pdf, you will need to tag each problem on the appropriate page of the document. Please show your work and draw a box around your final answer. You are free to work together with your classmates, but the work that you upload must be your own.

1. Consider the utility function $u(x_1, x_2) = x_1 + 2x_2$. Characterize the demand function for Good 1 if a consumer with this utility function maximizes utility subject to the linear budget constraint $p_1 x_1 + p_2 x_2 = m$.

   Hint: solve this problem by analyzing (i.e. staring at) the graph of the indifference curves and the budget line; your solution will have different cases (demand will be this if this is true, and demand will be this other thing if something else is true), like we saw in class when we looked at quasilinear utility.

2. Vilfredo's utility function is $u(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2}$, and he maximizes his utility subject to the budget constraint $p_1 x_1 + p_2 x_2 = m$.

   (a) Write down the Lagrangian for Vilfredo's utility maximization problem.

   (b) Find the derivative of $\mathcal{L}$ with respect to x_1, x_2, and λ.

   (c) Set the three derivatives equal to zero and solve this system of first-order conditions to arrive at Vilfredo's demand for Good 1.

3. Hardworking Harriet divides $L > 0$ hours of her time between work and leisure. ℓ indicates the number of hours of leisure. For her work, Harriet earns an hourly wage $w > 0$ which she immediately spends on a single consumption good. The price of a unit of the consumption good is normalized to one, so her budget constraint can be written as:

$$c = w\,(L - \ell)$$

   Her utility function over consumption and leisure is:

$$u\,(c, \ell) = \frac{-1}{c} + \frac{-1}{\ell}$$

   (a) Write down the Lagrangian for Harriet's utility maximization problem.

   (b) Find the derivatives of $\mathcal{L}$ with respect to ℓ, c, and λ.

   (c) Set the three derivatives equal to zero and solve this system of first-order conditions to arrive at Harriet's demand for leisure.

   Hint: consumption and leisure are always non-negative (which is important to remember when you take the square root of both sides of an equation).

4. Super Dad divides his Saturday between cleaning and laundry subject to his time budget constraint: the hours he spends on laundry and the hours he spends on cleaning must sum to $H > 0$, the number of hours in his day. If he cleans x rooms, it takes x^2 hours. If he does y loads of laundry, it takes y^2 hours. His utility function over cleaned rooms and completed loads of laundry is

$$u(x, y) = a \ln x + b \ln y$$

where $a > 0$ and $b > 0$ are positive constants. If Super Dad maximizes his utility and divides his Saturday between cleaning and laundry, how many rooms does he clean? Express your answer (Super Dad's "demand" for cleaning) in terms of a and b.

   (a) What is Super Dad's time budget constraint? There should not be any (explicit) prices in this equation.

   (b) Write down the Lagrangian for Super Dad's utility maximization problem.

   (c) Find the derivatives of $\mathcal{L}$ with respect to x, y, and λ.

   (d) Set the three derivatives equal to zero and solve this system of first-order conditions to arrive at Super Dad's demand for cleaning.

5. The constant elasticity of substitution (or CES) utility function can be written as:

$$u(x_1, x_2) = \frac{x_1^\delta}{\delta} + \frac{x_2^\delta}{\delta}$$

The parameter δ falls somewhere between $-\infty$ and 1. Wassily's preferences can be represented by a CES utility function, and he faces the budget constraint $p_1 x_1 + p_2 x_2 = m$. Solve Wassily's utility maximization problem to characterize his demand for Good 1. If you have done this correctly, you should be able to show that your demand function matches your solution to Problem 2 when the value of $\delta = 1/2$.

6. *This problem is challenging, and it is only worth one point. You may want to skip it.*

   Show that for an individual with a CES utility function with δ between $-\infty$ and 1 (see above) *expenditure* on Good 1, $p_1 x_1^* (p_1, p_2, m)$, **increases** as p_1 increases if $\delta < 0$ and **decreases** as p_1 increases if $\delta > 0$.

7. Which of your classmates did you work with on this problem set?

8. Did you attend Jamie's TA office hours, or get help from her over email or outside of her regular office hours?