ECON 251: Problem Set 4
Due Friday October 7 by 11:00 PM

Instructions. Each part of a problem is worth one point. Don't forget to answer the last two questions, as they are each worth one point. Unless otherwise stated, you can always assume that goods are continuously divisible: there is no need to consume an integer number of units of any good. Once you have finished, please submit your completed problem set on gradescope. To do this, you will either need to upload a pdf of your entire problem set or an image (for example, a picture that you take with your phone) of your work for each problem. If you upload a pdf, you will need to tag each problem on the appropriate page of the document. Please show your work and draw a box around your final answer. You are free to work together with your classmates, but the work that you upload must be your own.

1. The set AFC East contains four teams: the Bills, the Dolphins, the Jets, and the Patriots. Suppose they played the following games:

Team 1 (T1)	**Team 2 (T2)**	**T1 Score**	**T2 Score**	**Winner**
Dolphins	Bills	11	26	Bills
Dolphins	Patriots	17	16	Dolphins
Jets	Patriots	13	54	Patriots
Bills	Jets	45	17	Bills
Patriots	Bills	14	10	Patriots

   (a) How would you modify this table of game results so that the relation "defeated" (as in "Team A defeated Team B") is complete? In case this is not apparent, the team that with a higher score defeats the other team.

   (b) How would you modify this table of game results (the original table, without the modification that you made in part a) so that the relation "defeated" (as in "Team A defeated Team B") is transitive?

2. Suppose the preference relation $\succsim$ on $\boldsymbol{R}_+^2$ can be represented by the utility function $u(x_1, x_2) = \min\{x_1, x_2\}$.

   (a) Are preferences monotone?

   (b) Are preferences strongly monotone?

   (c) Are preferences convex?

   (d) Are preferences strictly convex?

3. Suppose the preference relation $\succsim$ on $\boldsymbol{R}_+^2$ can be represented by the utility function $u(x_1, x_2) = ax_1 + bx_2$ for some $a > 0$ and $b > 0$.

   (a) Are preferences monotone?

   (b) Are preferences strongly monotone?

(c) Are preferences convex?

(d) Are preferences strictly convex?

4. Suppose the preference relation $\succsim$ on $\boldsymbol{R}_+^2$ can be represented by the utility function $u(x_1, x_2) = -\sqrt{(x_1 - a)^2 + (x_2 - b)^2}$ for some $a > 0$ and $b > 0$.

   (a) Are preferences monotone?

   (b) Are preferences strongly monotone?

   (c) Are preferences convex?

   (d) Are preferences strictly convex?

5. Draw a graph of the indifference curves for the utility function $u(x_1, x_2) = \min\{2x_1, 4x_2\}$. Make sure to label it appropriately.

6. Suppose preferences can be represented by the utility function $u(x_1, x_2) = ax_1 + bx_2$. Calculate the marginal rate of substitution between Good 1 and Good 2.

7. Suppose preferences can be represented by the utility function $u(x_1, x_2) = x_1 + \sqrt{x_2}$. Calculate the marginal rate of substitution between Good 1 and Good 2.

8. Suppose preferences can be represented by the utility function $u(x_1, x_2) = x_1^\delta/\delta + x_2^\delta/\delta$. Calculate the marginal rate of substitution between Good 1 and Good 2.

9. Suppose preferences over Goods 1 through 100 can be represented by the utility function:

$$\begin{aligned} u(x_1, x_2, \ldots, x_100) &= a_1 \ln(x_1) + a_2 \ln(x_2) + \ldots + a_{100} \ln(x_{100}) \\ &= \sum_{i=1}^{100} a_i \ln(x_i) \end{aligned}$$

   where $a_1, a_2, \ldots, a_{100}$ are different positive constants. Calculate the marginal rate of substitution between Good 1 and Good 100.

10. Consider a decision maker whose preferences can be represented by the Cobb-Douglas utility function: $u(x_1, x_2) = x_1 x_2$. As discussed in class, the bundles $(3, 12)$ and $(9, 4)$ fall on the same indifference curve. Prove that any convex combination of these bundles falls on a higher indifference curve.

    As a reminder, given two bundles a and b comprising two goods (so $a = (a_1, a_2)$ and $b = (b_1, b_2)$ are both in $\boldsymbol{R}_+^2$), a convex combination of a and b is a bundle $c = (c_1, c_2)$ where

$$c_1 = \lambda a_1 + (1 - \lambda) b_1$$

    and

$$c_2 = \lambda a_2 + (1 - \lambda) b_2$$

    for some λ between 0 and 1.

11. Which of your classmates did you work with on this problem set?

12. Did you attend Jamie's TA office hours, or get help from her over email or outside of her regular office hours?