

ECON 251: Problem Set 10

Due Tuesday November 22 by 11:00 PM¹

Instructions. Each part of a problem is worth one point. Don't forget to answer the last two questions, as they are each worth one point. Unless otherwise stated, you can always assume that goods are continuously divisible: there is no need to consume an integer number of units of any good. Once you have finished, please submit your completed problem set on gradescope. To do this, you will either need to upload a pdf of your entire problem set or an image (for example, a picture that you take with your phone) of your work for each problem. If you upload a pdf, you will need to tag each problem on the appropriate page of the document. Please show your work and draw a box around your final answer. You are free to work together with your classmates, but the work that you upload must be your own.

1. Consider a producer with the production function $f(k, l) = k^a l^a$ for some a between 0 and one half ($0 < a < 1/2$), where $k > 0$ indicates the quantity of capital used in production and l indicates the quantity of labor used in production.
 - (a) Characterize the producer's Lagrangian as she wishes to minimize input costs when producing q units of output, given input prices r (the rental price of capital) and w (the wage).
 - (b) Calculate the producer's cost function $c(q)$.
 - (c) Given this cost function, find the profit-maximizing level of output, q^* if she is paid price p for every unit.
2. A profit-maximizing monopolist faces the linear demand function $Q(p) = a - bp$. Her marginal cost of production is constant: $MC(q) = \frac{\partial c(q)}{\partial q} = c$. The government imposes a tax of $t > 0$ on every unit sold, increasing the monopolist's marginal cost by t .
 - (a) Express the monopolist's marginal revenue as a function of her choice of q .
 - (b) How much does she choose to produce (to maximize her profits)?
 - (c) How much of the tax is passed on to the consumer? In other words, how much higher is the market price with the tax relative to the market price without the tax?
 - (d) Suppose that the same monopolist was instead seeking to maximize output subject to the constraint that profits be non-negative. How much of the tax would be passed on to the consumer in that case?
3. A market is made up nine (potential) buyers and six (potential) sellers. Each of the sellers is endowed with a single unit of an indivisible good, which they may or may not decide to sell. Each of the potential buyers is endowed with zero units of the indivisible good; each of the buyers is deciding whether or not to buy a single unit of the good, depending on the market price of the good $p > 0$.

Individual i 's utility depends on two things: (i) the amount of money (dollars) that she holds, $d_i \in \mathbf{R}_+$, and (ii) whether or not she owns one unit of the indivisible good, $x_i \in \{0, 1\}$. Individual i 's valuation of the indivisible good x is v_i , and i 's overall utility is given by:

$$u_i(d_i, x_i) = d_i + v_i x_i \tag{1}$$

¹You will not be marked late as long as you submit your problem set by 11:00 PM on Sunday, November 27.

So, if $x_i = 0$, utility is simply the number of dollars i has, d_i ; and if $x_i = 1$, utility is the sum of dollars held and individual i 's valuation of the indivisible good. Thus, buyers and sellers have similar utility functions, but sellers are endowed with a unit of the good ($x_i = 1$) which they may choose to sell if $p \geq v_i$, while buyers are not initially endowed with a unit of the good ($x_i = 0$). We can assume that for all buyers i , d_i is large enough that a buyer can purchase the good if she wishes to.

The valuations (i.e. the v_i values) of the nine buyers are: 1, 1, 2, 3, 5, 8, 13, 21, 34. The valuations (i.e. the v_i values) of the six sellers are: 2, 2, 4, 6, 10, 16. Find a whole number price that clears the market. Assume that if a buyer is indifferent between buying and selling, she buys; and if a seller is indifferent between buying and selling, she sells.

4. Suppose aggregate supply in a competitive market is characterized by the supply curve $S(p) = 10p$ and aggregate demand is characterized by the demand curve $D(p) = 100 - 15p$. What is the equilibrium price in this market?
5. Consider the competitive market described above. The supply curve is $S(p) = 10p$ and the demand curve is $D(p) = 100 - 15p$. The government imposes a tax of five dollars per unit purchased, such that when the buyer pays p_B , the seller only receives price $p_S = p_B - 5$. What is the equilibrium buyer's price p_B in this market?
6. Which of your classmates did you work with on this problem set?
7. Did you attend Jamie's TA office hours, or get help from her over email or outside of her regular office hours?