6 Labor Supply

6.1 Labor-Leisure Tradeoffs

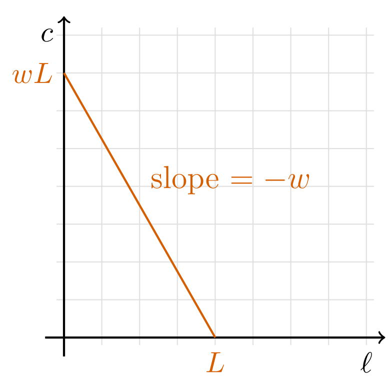
- A decision-maker is endowed with $L > 0$ hours
- They divide their time between hours of work, $h \geq 0$, and hours of leisure, $\ell \geq 0$
- Thus, hours worked are hours not spent on leisure:

$$h = L - \ell \tag{1}$$

- For every hour worked, the decision-maker earns wage w; so income if $wh = w(L - \ell)$
- Wages are spent on a single consumption good, and the price of a unit of the consumption good is normalized to $p = 1$
- We can write the budget constraint as

$$c \leq w\,(L - \ell) \Leftrightarrow c + w\ell \leq wL \tag{2}$$

- Individual utility depends on consumption, c, and leisure, ℓ
- Preferences can be represented by the utility function $u(c, \ell)$
- We can represent the decision-maker's budget set graphically:



- The scarce resource that the decision-maker is allocating is time, and a change in the wage affects both the value of the decision-maker's endowment (wL) and the opportunity cost of an hour of leisure (measured in foregone consumption)

6.2 Labor Supply and the Demand for Leisure

- Though the setup is different, we can solve the decision-maker's problem using our standard approach, writing down a Lagrangian and then differentiating with respect to c, ℓ, an λ

  - The Lagrangian:
$$\mathcal{L} = u(c, \ell) + \lambda (wL - c - w\ell) \tag{3}$$

  - The resulting first-order conditions:
    1. $\frac{\partial \mathcal{L}}{\partial c} = 0 \Leftrightarrow \frac{\partial u(c,\ell)}{\partial c} = \lambda$
    2. $\frac{\partial \mathcal{L}}{\partial \ell} = 0 \Leftrightarrow \frac{\partial u(c,\ell)}{\partial \ell} = \lambda w \Leftrightarrow \frac{1}{w} \cdot \frac{\partial u(c,\ell)}{\partial \ell} = \lambda$
    3. $\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Leftrightarrow c + w\ell = wL$

  - We can solve this system of equations for ℓ to find demand for leisure, $\ell^*(w, L)$; or for c to find demand for the consumption good, $c^*(w, L)$

  - We can also solve for labor supply: $h^*(w, L) = L - \ell^*(w, L)$

- **Example:** $u(c, \ell) = \sqrt{c} + \sqrt{\ell}$

  - The Lagrangian:
$$\mathcal{L} = \sqrt{c} + \sqrt{\ell} + \lambda (wL - c - w\ell) \tag{4}$$

  - The resulting first-order conditions:
    1. $\frac{\partial \mathcal{L}}{\partial c} = 0 \Leftrightarrow \frac{1}{2\sqrt{c}} = \lambda$
    2. $\frac{\partial \mathcal{L}}{\partial \ell} = 0 \Leftrightarrow \frac{1}{2\sqrt{\ell}} = \lambda w \Leftrightarrow \frac{1}{2w\sqrt{\ell}} = \lambda$
    3. $\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Leftrightarrow c + w\ell = wL$

- Combining FOC 1 and FOC 2:

$$\frac{1}{2\sqrt{c}} = \frac{1}{2w\sqrt{\ell}}$$
$$\Leftrightarrow \sqrt{c} = w\sqrt{\ell}$$
$$\Leftrightarrow c = w^2\ell$$

- Plugging this into FOC 3:

$$c + w\ell = wL$$
$$\Leftrightarrow w^2\ell + w\ell = wL$$
$$\Leftrightarrow \ell^*(w, L) = \frac{L}{1+w}$$

- Having solved for demand for the leisure, $\ell^*(w, L)$, we can easily solve for labor supply and the demand for consumption:

$$\begin{aligned} h^*(w, L) &= L - \ell^*(w, L) \\ &= L - \frac{L}{1+w} \\ &= \frac{L + wL}{1+w} - \frac{L}{1+w} \\ &= \frac{wL}{1+w} \end{aligned}$$

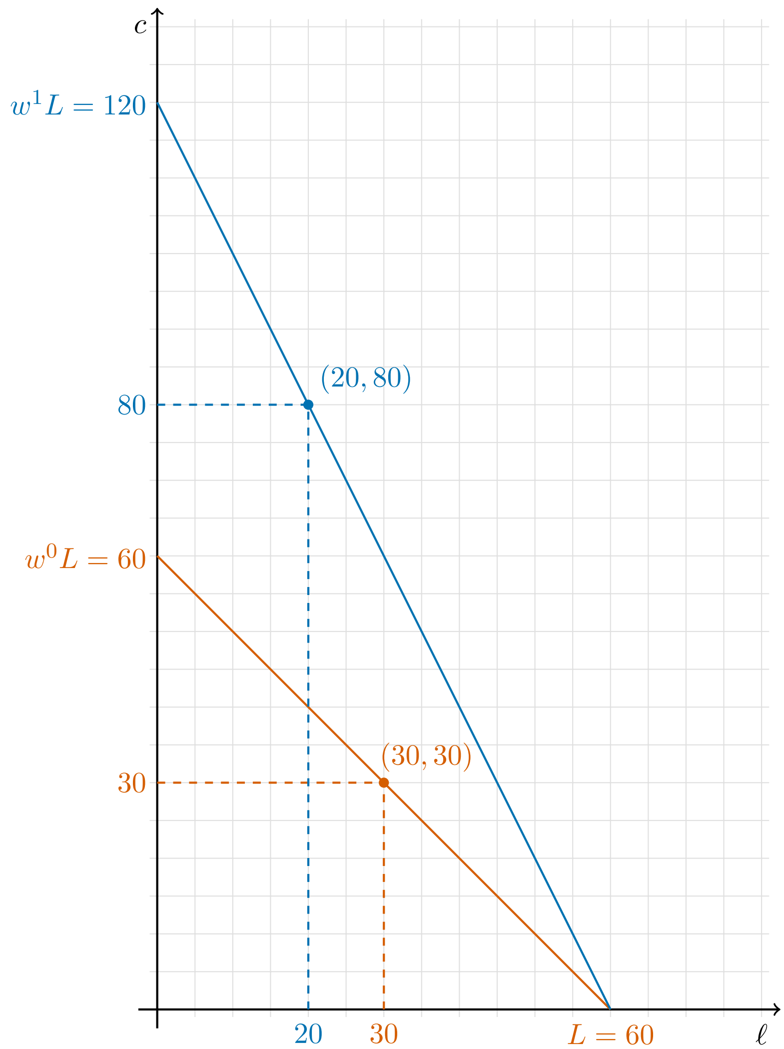
and

$$\begin{aligned} c^*(w, L) &= w^2\ell^*(w, L) \\ &= \frac{w^2 L}{1+w} \end{aligned}$$

- **Practice Problem:** find the demand for leisure, the demand for consumption, and labor supply if utility is $u(c, \ell) = \frac{-1}{c} + \frac{-1}{\ell}$
- In the examples above, does the demand for leisure increase or decrease as w increases?

6.3 Price Changes and Wealth Effects

- Consider the example above. Suppose $L = 60$. What happens when the wage increases from $w^0 = 1$ to $w^1 = 2$?

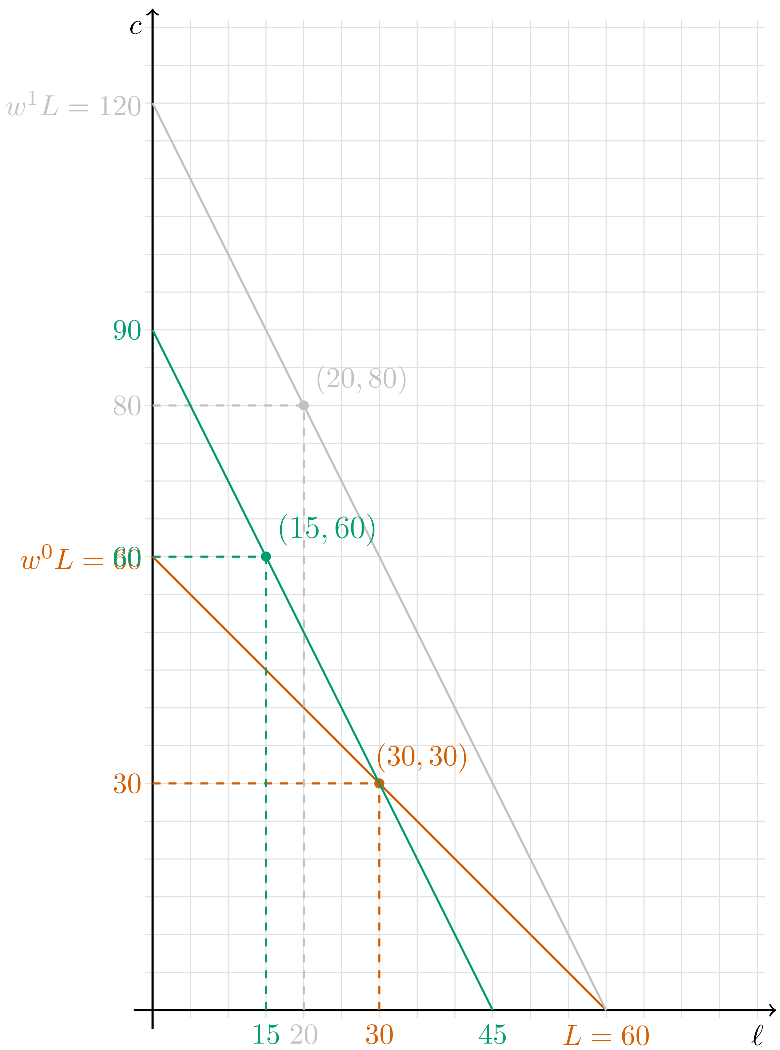


- When $L = 60$ and $w^0 = 1$, the decision-maker chooses the bundle $(30, 30)$ – 30 hours of leisure (and implicitly 30 hours of work) and 30 units of consumption

- When the wages rises to $w^1 = 2$, the decision-maker chooses $(20, 80)$ – 20 hours of leisure and 80 units of consumption

– Thus, the opportunity cost of leisure increases, and the demand for leisure decreases

- Now consider what would happen after a compensated price change – a shift to the new price ratio, but with a budget line that ran through the original chosen consumption bundle $(30, 30)$



- As you can see in the figure above, a compensated price change (from $w^0 = 1$ to $w^1 = 2$) would require a decrease in L, the endowment of hours, from 60 to 40

- After a compensated price change to $w^1 = 2$ and $L^c = 45$, the decision-maker would choose the bundle $(15, 60)$ (confirm this for yourself)
- Thus, as expected, an increase in the opportunity cost of leisure leads to a substitution away from leisure after a compensated price change (again, the substitution effect is non-positive)
- What is more surprising is that the actual price – an increase in the implicit price of leisure – change involves shifting the budget line *out* from the compensated price change; we would normal expect an increase in the price of one of the goods to be equivalent to shifting the budget line in instead of out
- In this case, the value of the endowment also depends on w, so an increase in the wage raises the opportunity cost of leisure and the overall value of the decision-maker's endowment
- This gives rise to a **wealth effect**, a change in demand that results from a change in the value of an individual's assets
- Intuitively, because w impacts both the opportunity cost of choosing ℓ hours of leisure and the overall value of the endowment, the change in w is mathematically equivalent to a change in the price of the consumption good
- Because of wealth effects, we will sometimes see demand for leisure *increase* after an increase in w, even when leisure is not a Giffen good
- You can see this will be true when utility is $u(c, \ell) = \frac{-1}{c} + \frac{-1}{\ell}$:
  - Consider a price rise from $w^0 = 1$ to $w^1 = 4$ (so that the math works out nicely)
  - You should be able to show that the substitution effect is negative, but the overall change in demand for leisure is positive – because the increase in the value of the endowment has made the decision-maker wealthier

6.4 Government Policy and Labor Supply

- Government policies impact the shape of the budget set over consumption and leisure:
  - Social protection (i.e. transfer) programs

  - Income taxes
  - Laws about overtime pay
  - Restrictions on hours

- Employers also place restrictions on the budget set, for example by requiring people to work a minimum number of hours per week

6.4.1 Government Transfers and Non-Labor Income

- Consider a simply extension to the model described above: individuals receive transfer income $\tau > 0$ from the government, in addition to their labor income
  - If an individual works zero hours, they can consume up to τ units of the consumption good
  - If they work L hours, their total income is $wL + \tau$

- The point of tangency between the budget line and an indifference curve may occur at $\ell > L$, which is impossible because the decision-maker cannot consume more than L hours of leisure; this would occur if the decision-maker preferred more than L hours of leisure and less than τ units of consumption over all available consumption bundles

- In this case, the highest attainable level of utility will occur at $\ell = L$: the decision-maker will choose not to work at all, and will fund consumption with transfer income

6.4.2 Income Taxes

- Graduated income taxes can also create kinked budget lines

- If the tax rate is t_{low} for low levels of income and $t_{high} > t_{low}$ for higher levels of income, the slope of the budget line will be flatter for relatively low levels of leisure