4 Preferences and Utility

4.1 Preferences

- Some "prefer" statements:
  - "I prefer onions on my pizza"
  - "I prefer to work out at the gym"
  - "I prefer that blue Subaru Forester over that black Outback"
- Only one of those sentences illustrates a preference as economists use the term
  - Saying that you prefer onions does not actually make it clear how you rank pizzas. Are all pizzas with onions better than all pizzas without onions? Are all pizzas with onions equally good. When economists talk about preferences, we are talking about preferring one element in a choice set (i.e. one consumption bundle) over another, not making a statement about attributes that consumption bundles might have
  - When you say that you prefer to work out at the gym, the question is: over what? Over going for a run? Over receiving an all expense paid trip to Hawaii?
- When economists talk about preferences, we are talking about how an individual compares, or ranks, two possible consumption bundles.
  - We don't usually say "Adam prefers Bundle A" unless he prefers it over all other bundles (and even then we say "it is most preferred" or something)
  - We say "Adam prefers Bundle A to Bundle B"
  - Exactly analogous to $\geq$ with numbers: you can't just say "$x \geq$"
- A **preference relation** or **preference ordering** $\succsim$ is a ranking of bundles that is complete and transitive
  - A ranking is **complete** if for any two consumption bundles a and b, either $a \succsim b$ or $b \succsim a$ (or both could be true)
  - A ranking is **transitive** if $a \succsim b$ and $b \succsim c$ together imply $a \succsim c$

- An individual is **rational** if they have (complete and transitive) preferences
- Intuitively, a preference ordering is a sorting of consumption bundles into ranked bins of equivalent bundles, from most preferred to least preferred
  - This is about math, not about tastes: preferences do not have to make sense in any meaningful way, and we can define a preference ordering over an arbitrary set of bundles $X = \{a, b, c, d\}$
- From a preference relation, we can define two other types of relations:
  - $a \sim b \Leftrightarrow a \succsim b$ and $b \succsim a$ (indifference)
  - $a \succ b \Leftrightarrow a \succsim b$ and $b \not\succsim a$ (strict preference)

4.2 Utility Functions

- A **utility function** maps bundles to an ordinal index of their relative attractiveness
  - In other words, a utility function assigns each equivalence class a number, with more preferred indifference sets receiving higher numbers
  - The numbers don't matter, only the ranking: you can multiply any utility function by a positive constant, or take any increasing transformation, and it still gives you the same ordering of indifference sets
- A utility function $u(x)$ **represents** a preference ordering $\succsim$ if and only if for any two consumption bundles a and b, $a \succsim b \Leftrightarrow u(a) \geq u(b)$
  - This definition implies that $a \succ b \Leftrightarrow u(a) > u(b)$ and $a \sim b \Leftrightarrow u(a) = u(b)$
  - If the choice set is finite (e.g. $X = \{a, b, c, d\}$), then completeness and transitivity are enough to guarantee that a utility representation exists
  - Unfortunately, completeness and transitivity are **not** enough to guarantee that a utility function exists to represent preferences on choice sets that contain an infinite number of bundles (like our typical budget sets)
- **Counterexample: lexicographic preferences**

  - let $x, \in \boldsymbol{R}_{+}^{2}$ be consumption bundles
  - Define preference relation $\succsim$ such that $x \succsim y$ if $x_1 > y_1$ or if $x_1 = y_1$ and $x_2 \geq y_2$
  - In words: x is weakly preferred to y if x contains more of Good 1 than y does, or if x and y contain equal amounts of Good 1 and x has at least as much of Good 2 as y
  - This is where the name comes from: the ordering is like the ordering of words in the dictionary
  - *Draw the graph*
  - Indifference sets are singletons: no two bundles are equally good
  - Worse than that, it takes all the real numbers to rank the relative attractiveness of the bundles that contain a fixed amoung of Good 1, but to order all the possible bundle we'd need to do that infinity times
  - There is no way to aggregate the information on the amounts of the different goods into a single index of the attractiveness of a bundle
  - Intuitively, the reason is that with lexicographic preferences, there is no willingness to make a tradeoff across goods and how much you like a bundle depends on the amount of both goods in the bundle
  - Going back to the name: why do dictionaries work? Because there aren't infinity letters in the alphabet.

- To guarantee a utility representation, we need a preferences to have a third property
- A preference ordering $\succsim$ on $\boldsymbol{R}_{+}^{k}$ is **continuous** if $x \succsim y$ implies that bundles that are sufficiently close to x are also weakly preferred to bundles that are sufficiently close to y
- *Draw a graph, and then return to the graph of lexicographic preferences to show that they are not continuous*
- If a preference relation $\succsim$ on $\boldsymbol{R}_{+}^{k}$ is rational (complete and transitive) and continuous, then a utility function $u(x)$ exists that represents $\succsim$

4.3 Indifference Curves

- Given a preference relation $\succsim$ on $\boldsymbol{R}_+^2$, we can plot the indifference sets (which we often refer to as **indifference curves** when they are not thick)
  - *Draw some graphs*
  - They could be sets and not curves
  - Better could mean less when the goods are actually bads
- When $\succsim$ has a utility representation $u(x)$, indifference curves are just graphs of the level sets of the utility function: $\left\{x \in \boldsymbol{R}_+^k : u(x) = c\right\}$ for some constant c
  - This highlights the fact that the numbers don't matter: if you instead used the utility function $v(x) = \ln(u(x))$ or $v(x) = a \cdot u(x) + b$), you'd get exactly the same level sets

4.4 Canonical Examples

- Perfect substitutes: $u(x_1, x_2) = x_1 + x_2$ or more generally $u(x_1, x_2) = ax_1 + bx_2$ for some $a > 0$ and $b > 0$
- Perfect complements: $u(x_1, x_2) = \min\{x_1, x_2\}$ or more generally $u(x_1, x_2) = \min\{ax_1, bx_2\}$ for some $a > 0$ and $b > 0$
- Selfish preferences: $u(x_1, x_2) = f(x_1)$
- Optimal bundle: $u(x_1, x_2 x) = -\sqrt{(x_1 - a)^2 + (x_2 - b)^2}$
- Cobb-Douglas: $u(x_1, x_2) = \ln(x_1) + \ln(x_2)$ or more generally $u(x_1, x_2) = a\ln(x_1) + b(x_2)$ for some $a > 0$ and $b > 0$
  - Notice that we could also represent Cobb-Douglas as $u(x) = x_1 x_2$

4.5 The Marginal Rate of Substitution

- The **marginal rate of substitution** (MRS) is minus one times the slope of the indifference curve at a particular point

  - How much of Good 2 would be willing to give up to receive a small increase in Good 1 (at the margin, not an entire additional unit of Good 1)

- **Example: Calculating the MRS with Cobb-Douglas Utility**
  - *Graph Cobb-Douglas indifference curves, show that* $(3,12)$ *and* $(9,4)$ *are on the same indifference curve*
  - Solve for MRS by explicitly solving equation characterizing indifference curve for x_2, differentiating
  - Calculate MRS at $(3,12)$ and $(9,4)$
  - Implicit Function Theorem
  - Solve for MRS using the Implicit Function Theorem
  - Confirm that answer match above for $(3,12)$ and $(9,4)$

4.6 Properties of Preferences

- A preference relation $\succsim$ on $\boldsymbol{R}^2_+$ is **monotone** both of the following properties hold:

  (i) If $x_1 \geq y_1$ and $x_2 \geq y_2$, then $(x_1, x_2) \succsim (y_1, y_2)$

  (ii) If $x_1 > y_1$ and $x_2 > y_2$, then $(x_1, x_2) \succ (y_1, y_2)$

- A preference relation $\succsim$ on $\boldsymbol{R}^2_+$ is **strongly monotone** if $x_1 \geq y_1$, $x_2 \geq y_2$, and $x \neq y$ together imply $(x_1, x_2) \succ (y_1, y_2)$
  - If $x_1 \geq y_1$, $x_2 \geq y_2$, and $x \neq y$, then it must be the case that $x_1 > y_1$ or $x_2 > y_2$
  - Strong monotonicity is different form non-strong monotonicity because it requires bundles to be preferred when they contain more of any one good; under non-strong monotonicity, a bundle is preferred if it contains more of both goods
  - Strong monotonicity implies regular monotonicity, but not vice versa
  - A preference relation $\succsim$ on $\boldsymbol{R}^2_+$ is **convex** if for any two bundles a and b,

$$a \succsim b \Rightarrow \lambda a + (1 - \lambda)b \succsim b$$

    for any λ between 0 and 1

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$$a \succsim b \Rightarrow \lambda a + (1-\lambda) b \succ b$$

  for any λ between 0 and 1

- Convexity means that the upper contour sets of the utility function are convex: in other words, for any bundle x, the set of bundles that are weakly preferred ot x is a convex set